INTERNATIONAL LIQUIDITY AND EXCHANGE RATE DYNAMICS*

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We provide a theory of the determination of exchange rates based on capital flows in imperfect financial markets. Capital flows drive exchange rates by altering the balance sheets of financiers that bear the risks resulting from international imbalances in the demand for financial assets. Such alterations to their balance sheets cause financiers to change their required compensation for holding currency risk, thus affecting both the level and volatility of exchange rates. Our theory of exchange rate determination in imperfect financial markets not only helps rationalize the empirical disconnect between exchange rates and traditional macroeconomic fundamentals, it also has real consequences for output and risk sharing. Exchange rates are sensitive to imbalances in financial markets and seldom perform the shock absorption role that is central to traditional theoretical macroeconomic analysis. Our framework is flexible; it accommodates a number of important modeling features within an imperfect financial market model, such as nontradables, production, money, sticky prices or wages, various forms of international pricing-to-market, and unemployment. JEL Codes: E44, F31, F32, F41, G11, G15, G20.

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I. Introduction

We provide a theory of exchange rate determination based on capital flows in imperfect financial markets. In our model, exchange rates are governed by financial forces because global shifts in the demand and supply of assets result in large-scale capital flows that are intermediated by the global financial system. The demand and supply of assets in different currencies and the willingness of the financial system to absorb the resulting imbalances are first-order determinants of exchange rates. Despite extensive debates on these financial forces and their implications for exchange rates, there are very few tractable frameworks to provide a unified analysis of such phenomena.

Active risk taking in currency markets is highly concentrated in few large financial players. These institutions range from the (former) proprietary desks and investment management arms of global investment banks, such as Goldman Sachs and JP Morgan, to macro and currency hedge funds such as Soros Fund Management to active investment managers and pension funds such as PIMCO and BlackRock. Although there are certainly significant differences across these intermediaries, we stress their common characteristic of being active investors that profit from medium-term imbalances in international financial markets, often by bearing the risks (taking the other side) resulting from imbalances in currency demand due to both trade and financial flows. They also share the characteristic of being subject to financial constraints that limit their ability to take positions, based on their risk-bearing capacities and existing balance sheet risks.

Our model captures this element of reality by placing financiers at the core of exchange rate determination. In our model, financiers absorb a portion of the currency risk originated by

1. Detailed data on risk taking in this international and opaque over-the-counter market are relatively scarce, particularly since a number of players, such as hedge funds, have low reporting requirements. It is precisely this nature of the market that favors specialization and concentration. Transaction volume data, however, also portray a highly concentrated market. The top 10 banks accounted for 80 percent of all flows in 2014, with the top two banks (Citigroup and Deutsche Bank) accounting for 32 percent of all flows (Euromoney 2014). Not only are these institutions large players in currency markets, currency risk also accounts for a large fraction of their overall respective risk taking. Regulatory filings reveal that currency risk accounted for 26–35 percent of total (stressed) value at risk at Deutsche Bank in 2013 and between 17 percent and 23 percent at Citigroup in the same period (Deutsche Bank 2013; Citigroup 2013).
imbalanced global capital flows. Alterations to the size and composition of financiers’ balance sheets induce them to differentially price currency risk, thus affecting both the level and the volatility of exchange rates. Our theory of exchange rate determination in imperfect financial markets differs from the conventional open macroeconomic model by introducing financial forces, such as portfolio flows, financiers’ balance sheets, and risk-bearing capacity, as first-order determinants of exchange rates.

A number of stylized facts have emerged from the empirical analysis of international financial markets: the failure of the uncovered interest rate parity condition (UIP) and the associated profitable carry trade, the presence of large-scale global (gross) capital flows putting appreciation pressure on the currencies of inflow-recipient countries, the disconnect of exchange rates from macro fundamentals, the vulnerability of external-debtor countries’ currencies to global financial shocks, and the impact of large-scale currency interventions by governments. At the same time the global financial crisis has highlighted the importance of financial frictions not only for outcomes in financial markets but also for real outcomes such as output and risk sharing. The main purpose of this article is to provide a tractable framework to jointly analyze these issues (some classic, some new) and provide a number of new insights.

Financiers actively trade currencies but have limited risk-bearing capacity: in equilibrium, a global imbalance that requires financiers to be long a currency generates an increase in the expected return of this currency. This has to occur to provide incentives to financiers to use part of their limited risk-bearing capacity to absorb the imbalance. All else equal, the currency has to depreciate today and be expected to appreciate in the future for financiers to earn compensation for their risk taking. This is the central exchange rate determination mechanism in the model.

Based on this framework, we analyze the importance of capital flows, that is, demand for foreign currency–denominated assets, in directly determining exchange rates. Whenever they are not matched globally, these global flows generate an imbalance and, via the constraints of the financiers, a direct effect on both the level and dynamics of the exchange rate. Consequently, countries that have recently received capital inflows tend to have risky currencies that depreciate if financiers’ risk-bearing capacity is disrupted. Since these countries have borrowed from financiers, their currencies in equilibrium have high expected returns
to incentivize financiers to lend. A financial disruption, by reducing financiers’ risk-bearing capacity, generates an immediate currency depreciation and an expectation of future currency appreciation to increase financiers’ incentive to sustain the imbalance.

The model accounts for the failure of the UIP and provides a financial view of the carry trade whereby the trade performs poorly whenever adverse shocks to the financial system occur. UIP is violated because the financiers’ limited risk-bearing capacity induces them to demand a currency risk premium. In this world the carry trade is profitable because, given an interest rate differential, financiers’ limited risk-bearing capacity precludes them from taking enough risk to completely exploit the profitability of the trade. Similarly, financial disruptions generate a need to increase the expected returns of the carry trade: this is achieved with an immediate loss in the carry trade and an expectation of its recovery going forward.

The exchange rate is disconnected from traditional macroeconomic fundamentals, such as imports, exports, consumption, and output, in as much as these same fundamentals correspond to different equilibrium exchange rates depending on financiers’ balance sheets and risk-bearing capacity. Financiers act as shock absorbers, by using their risk-bearing capacity to accommodate flows that result from fundamental shocks, and are themselves the source of financial shocks that distort exchange rates.

The financial determination of exchange rates in imperfect financial markets has real consequences for output and risk sharing. To more fully analyze these consequences, we extend the basic model by introducing a simple model of production under both flexible and sticky prices. For example, in the presence of goods’ prices that are sticky in the producers’ currencies, a capital inflow or financial shock that produces an overly appreciated exchange rate causes a fall in demand for the inflow-receiving country’s exports and a corresponding fall in output. This perverse effect of capital flows transmits frictions from financial markets to the real economy.

In our model, currency intervention by the government is effective because, as a capital flow, it alters the balance sheet of constrained financiers. The potency of the intervention relies entirely on the frictions; there would be no effect from the intervention absent financial imperfections. We show that a commonly adopted policy combination of currency intervention and capital...
controls can be understood as capital controls exacerbating financial imperfections, thus further segmenting the currency market and increasing the potency of currency intervention. We show that if a country has an overly appreciated currency and its output is demand-driven, that is, output would increase via an increase in exports if the currency were to depreciate, then a currency intervention increases output at the cost of distorting consumption risk-sharing intertemporally.

Throughout the article we stress tractability and make simplifying functional assumptions that make our model a convenient specification. We believe the simple modeling offers a number of insights with pencil-and-paper analysis. Of course, we also appreciate the need for optimization and general equilibrium, both of which are present in our set-up. However, we leave for the Appendix and in large part to future research to provide deeper contracting foundations for the frictions that we study and to assess in a large-scale model their quantitative implications.

This article is related to three broad streams of literature: early literature on portfolio balance, literature on portfolio demand in complete or incomplete markets, and literature on frictions and asset demand. Our work is inspired by a number of ideas in the early literature on portfolio balance models by Kouri (1976) and Driskill and McCafferty (1980a). A number of prominent economists have lamented that this earlier research effort “had its high watermark and to a large extent a terminus in Branson and Henderson (1985) handbook chapter” (see Obstfeld 2004) and is “now largely and unjustly forgotten” (see Blanchard, Giavazzi, and Sa 2005). The literature that followed this earlier modeling effort has either focused on UIP-based analysis (Obstfeld and Rogoff 1995) or mostly focused on currency risk premia in complete markets (Lucas 1982; Backus, Kehoe, and Kydland 1992; Backus and Smith 1993; Dumas 1992; Verdelhan 2010; Colacito and Croce 2011; Hassan 2013). Pavlova and Rigobon (2007) analyze a real model with complete markets where countries’


3. Among others see also Farhi and Gabaix (2014), Martin (2011), and Stathopoulos (2012).
representative agents have logarithmic preferences affected by taste shocks similar to those considered in this article. A smaller literature has analyzed the importance of incomplete markets (for recent examples: Chari, Kehoe and McGrattan 2002; Corsetti, Dedola, and Leduc 2008; Pavlova and Rigobon 2012).

The most closely related stream of the literature is the small set of papers that focused on exchange rate modeling in the presence of frictions. One important set of papers by Jeanne and Rose (2002), Evans and Lyons (2002), Hau and Rey (2006), Bruno and Shin (2014) studies frictions in financial markets in the absence of a real side of the economy with production, imports, and exports. One other important set of publications has a very different focus: informational frictions, infrequent portfolio rebalancing, or frictions in access to domestic money/funding market. Evans and Lyons (2012) focus on how disaggregate order flows from customers might convey information about the economy fundamentals to exchange rate market makers who observe the consolidated flow. Bacchetta and Van Wincoop (2010) study the implications of agents that infrequently rebalance their portfolio in an overlapping generations setting. Alvarez, Atkeson, and Kehoe (2002, 2009) and Maggiori (2014) are models of exchange rates in which the frictions, a form of market segmentation, are only present in the domestic money market or funding market.

II. BASIC GAMMA MODEL

Let us start with a minimalistic model of financial determination of exchange rates in imperfect financial markets. This simple real model carries most of the economic intuition and core modeling that we will extend to more general set-ups.

Time is discrete and there are two periods: $t = 0, 1$. There are two countries, the United States and Japan, each populated by a continuum of households. Households produce, trade (internationally) in a market for goods, and invest with financiers in risk-free bonds in their domestic currency. Financiers

4. Similar preferences are also used in Pavlova and Rigobon (2008, 2010).
5. In the absence of a nominal side to the model, which we add in Section IV, we intentionally abuse the word currency to mean a claim to the numéraire of the economy, and exchange rate to mean the real exchange rate. Similarly we abuse the words dollar- or yen-denominated to mean values expressed in units of nontradable goods in each economy.
intermediate the capital flows resulting from households’ investment decisions. The basic structure of the model is displayed in Figure I.

Intermediation is not perfect because of the limited commitment of the financiers. The limited-commitment friction induces a downward-sloping demand curve for risk taking by financiers. As a result, capital flows from households move financiers up and down their demand curve. Equilibrium is achieved by a relative price, in this case the exchange rate, adjusting so that international financial markets clear given the demand and supply of capital denominated in different currencies. In this sense, exchange rates are financially determined in an imperfect capital market.

We now describe each of the model’s actors, their optimization problems, and analyze the resulting equilibrium.

II.A. Households

Households in the United States derive utility from the consumption of goods according to:

\[ \theta_0 \ln C_0 + \beta E[\theta_1 \ln C_1], \]

where \( C \) is a consumption basket defined as:

\[ C_t = \left[ (C_{NT,t})^{\alpha_t} (C_{H,t})^{\alpha_t} (C_{F,t})^{\mu_t} \right]^{1/\gamma}, \]

where \( C_{NT,t} \) is the U.S. consumption of its nontradable goods, \( C_{H,t} \) is the U.S. consumption of its domestic tradable goods, and \( C_{F,t} \) is the U.S. consumption of Japanese tradable goods. We
use the notation \( \{x_t, a_t, u_t\} \) for nonnegative, potentially stochastic preference parameters and we define \( \theta_t \equiv x_t + a_t + u_t \). The nontradable good is the numéraire in each economy and, consequently, its price equals 1 in domestic currency (\( p_{NT} = 1 \)).

Households can trade both tradable goods in a frictionless goods market across countries, but can only trade nontradable goods within their domestic country. Financial markets are incomplete, and each country trades a risk-free domestic currency bond. The assumption that each country only trades in its own currency bonds is made here for simplicity and to emphasize the currency mismatch that the financiers have to absorb; we relax the assumption in later sections. Risk-free here refers to paying one unit of nontradable goods in all states of the world and is therefore akin to “nominally risk-free.”

U.S. households’ optimization problem is:

\[
\max_{\{C_{NT,t}, C_{H,t}, C_{F,t}\}} \theta_0 \ln C_0 + \beta \mathbb{E}[\theta_1 \ln C_1],
\]

subject to equation (2), and

\[
\sum_{t=0}^{\infty} R^{-t}(Y_{NT,t} + p_{H,t}Y_{H,t}) = \sum_{t=0}^{\infty} R^{-t}(C_{NT,t} + p_{H,t}C_{H,t} + p_{F,t}C_{F,t}).
\]

U.S. households maximize the utility by choosing their consumption and savings in dollar bonds subject to the state-by-state dynamic budget constraint. The households’ optimization problem can be divided into two separate problems. The first is a static problem, whereby households decide, given their total consumption expenditure for the period, how to allocate resources to the consumption of various goods. The second is a dynamic problem, whereby households decide intertemporally how much to save and consume.

The static utility maximization problem takes the form:

\[
\max_{\{C_{NT,t}, C_{H,t}, C_{F,t}\}} x_t \ln C_{NT,t} + a_t \ln C_{H,t} + u_t \ln C_{F,t}
\]

\[
+ \lambda_t (CE_t - C_{NT,t} - p_{H,t}C_{H,t} - p_{F,t}C_{F,t}),
\]

where \( CE_t \) is aggregate consumption expenditure, which is taken as exogenous in this static optimization problem and later endogenized in the dynamic optimization problem, \( \lambda_t \) is the associated Lagrange multiplier, \( p_{H,t} \) is the dollar price in the United States of U.S. tradables, and \( p_{F,t} \) is the dollar price
in the United States of Japanese tradables. First-order conditions imply $\frac{\lambda_{t}}{C_{NT,t}} = \lambda_{t}$, and $\frac{\lambda_{t}}{p_{F,t}} = \lambda_{t}p_{F,t}$. We assume that nontradable goods are produced by an endowment process that for simplicity follows $Y_{NT,t} = \chi_{t}$, unless otherwise stated. This simplifying assumption, combined with the market clearing condition for nontradables $Y_{NT,t} = C_{NT,t}$, implies that in equilibrium $\lambda_{t} = 1$ in all states. The assumption, although stark, makes the analysis of the basic model most tractable by neutralizing variations in household marginal utility that are not at the core of our article. The neutralization occurs because variation in household marginal utility is stabilized by a proportionate adjustment in the consumption of the nontradable good.\textsuperscript{6} With this assumption in hand, the dollar value of U.S. imports is:

$$p_{F,t}C_{F,t} = \iota_{t}.$$

Japanese households derive utility from consumption according to $\theta_{0}^{*}\ln C_{0}^{*} + \beta^{*}[E[\theta_{t}^{*}\ln C_{t}^{*}]]$, where starred variables denote Japanese quantities and prices. By analogy with the U.S. case, the Japanese consumption basket is:

$$C_{t}^{*} = \left[\left(C_{NT,t}^{*}\right)^{\alpha_{t}}(C_{H,t}^{*})^{\xi_{t}}\right]^{\frac{1}{\alpha_{t}}}.$$

The Japanese static utility maximization problem, reported for brevity in the Online Appendix, together with the assumption $Y_{NT,t}^{*} = \chi_{t}^{*}$, leads to a yen value of U.S. exports to Japan, $p_{H,t}^{*}C_{H,t}^{*} = \xi_{t}$, that is entirely analogous to the import expression derived above.

The exchange rate $e_{t}$ is defined as the quantity of dollars bought by ¥1, that is, the strength of the yen. Consequently, an increase in $e$ represents a dollar depreciation.\textsuperscript{7} The dollar value of U.S. exports is $e_{t}\xi_{t}$. U.S. net exports, expressed in dollars, are given by $NX_{t} = e_{t}p_{H,t}^{*}C_{H,t}^{*} - p_{F,t}C_{F,t} = \xi_{t}e_{t} - \iota_{t}$.\textsuperscript{8} We collect these results in the Lemma below.

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\textsuperscript{6} We stress that the assumption is one of convenience, and not necessary for the economics of the article. Online Appendix A.4 provides more general results that do not impose this assumption.

\textsuperscript{7} In this real model, the exchange rate is related to the relative price of nontradable goods. Online Appendix A.1.D provides a detailed discussion of different exchange rate concepts in this economy, including the nominal and CPI-based real exchange rate.

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Lemma 1. (Net Exports). Expressed in dollars, U.S. exports to Japan are \(\xi_t \varepsilon_t\); U.S. imports from Japan are \(\eta_t\); so that U.S. net exports are \(NX_t = \xi_t \varepsilon_t - \eta_t\).

Note that this result is independent of the pricing procedure (e.g., price stickiness under either producer or local currency pricing). Under producer currency pricing (PCP) and in the absence of trade costs, the U.S. dollar price of Japanese tradables is \(\frac{p_H}{e}\), while under local currency pricing (LCP) the price is simply \(p_H^e\).

It follows that under financial autarky, that is, if trade has to be balanced period by period, the equilibrium exchange rate is \(e_t = \frac{\xi_t}{\varepsilon_t}\). In financial autarky, the dollar depreciates (\(\uparrow e\)) whenever U.S. demand for Japanese goods increases (\(\uparrow \eta\)) or whenever Japanese demand for U.S. goods falls (\(\downarrow \xi\)). This has to occur because there is no mechanism, in this case, to absorb the excess demand/supply of dollars versus yen that a nonzero trade balance would generate.

The optimization problem (3) for the intertemporal consumption-saving decision leads to a standard optimality condition (Euler equation):

\[
1 = \mathbb{E}\left[\beta R \frac{U_t'_{C_{NT}}} {U_0'_{C_{NT}}} \right] = \mathbb{E}\left[\beta R \frac{(\frac{\ln(\xi_t)}{C_{NT,t}})} {(\frac{\ln(\varepsilon_t)}{C_{NT,0}})} \right] = \beta R,
\]

where \(U_t'_{C_{NT}}\) is the marginal utility at time \(t\) over the consumption of nontradables. Given our simplifying assumption that \(C_{NT,t} = \chi_t\), the Euler equation implies that \(R = \frac{1}{\beta}\). An entirely similar derivation yields \(R^* = \frac{1}{\beta}\). It might appear surprising that in a model with risk-averse agents the equilibrium interest rate equals the rate of time preference. Of course, this occurs here because the marginal utility of nontradable consumption, in which the bonds are denominated, is constant in equilibrium given the assumption \(C_{NT,t} = \chi_t\); so that the precautionary and intertemporal consumption smoothing desires simplify to be exactly zero.

We stress that the aim of our simplifying assumptions is to create a real structure of the basic economy that captures the main forces (demand and supply of goods), while making the real side of the economy as simple as possible. This allows us to analytically flesh out the crucial forces of the article in the
financial markets in the next sections without carrying around a burdensome real structure. Should the reader be curious as to the robustness of our model to relaxing some of the assumptions made so far, the quick answer is that it is quite robust.\(^9\)

### II.B. Financiers

Suppose that global financial markets are imbalanced, such that there is an excess supply of dollars versus yen resulting from, say, trade or portfolio flows. Who will be willing to absorb such an imbalance by providing Japan those yen and holding those dollars? We posit that the resulting imbalances are absorbed, at some premium, by global financiers.

We assume that there is a unit mass of global financial firms, each managed by a financier. Agents from the two countries are selected at random to run the financial firms for a single period.\(^10\) Financiers start their jobs with no capital of their own and can trade bonds denominated in both currencies. Therefore, their balance sheet consists of \(q_0\) dollars and \(-\frac{q_0}{e_0}\) yen, where \(q_0\) is the dollar value of dollar-denominated bonds the financier is long of and \(-\frac{q_0}{e_0}\) the corresponding value in yen of yen-denominated bonds. At the end of (each) period, financiers pay their profits and losses out to the households.\(^11\)

We assume that each financier maximizes the expected value of her firm:

\[
V_0 = \mathbb{E}\left[\beta \left(R - R^* \frac{e_1}{e_0}\right)\right] q_0 = \Omega_0 q_0.
\]

This valuation of currency trading corresponds to that of the household if they were allowed to trade optimally in foreign

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9. We make such robustness explicit in the Online Appendix.

10. In this setup, being a financier is an occupation for agents in the two countries rather than an entirely separate class of agents. The selection process is governed by a memoryless Poisson distribution. Of course, there are no selection issues in the one-period basic economy considered here, but we proceed to describe a more general setup that will also be used in the model extensions.

currency. Indeed, if U.S. households were allowed to trade optimally yen bonds as well as dollar bonds we would recover the standard Euler equation:

\[
0 = \mathbb{E}\left[ \beta \frac{U_{1,C_{NT}}}{U_{0,C_{NT}}}(R - R^* \frac{e_1}{e_0}) \right] = \mathbb{E}\left[ \beta \frac{x_1}{C_{NT,t}} (R - R^* \frac{e_1}{e_0}) \right] = \mathbb{E}\left[ \beta (R - R^* \frac{e_1}{e_0}) \right],
\]

where the last equality follows from the assumption that \( C_{NT,t} = x_t \) and the result that \( \beta R - 1 \) derived previously (see equation (6)). Households optimally value the currency trade according to its expected (discounted at \( R \)) excess returns. Notice that this mean-return criterion holds despite the households being risk-averse. The simplification occurs because variations in marginal utility are exactly offset by variations in the relative price of nontradable goods, so that marginal utility in terms of the numéraire (the NT good) is constant across states of the world. In the absence of frictions our financiers would simply be a veil and the optimality condition in maximization (7) would impose the household optimality criterion:

\[
0 = \mathbb{E}\left[ \beta (R - R^* \frac{e_1}{e_0}) \right].
\]

To capture the role of limited financial risk-bearing capacity by the financiers, we assume that in each period, after taking positions but before shocks are realized, the financier can divert a portion of the funds she intermediates. If the financier diverts the funds, her firm is unwound and the households that had lent to her recover a portion \( 1 - \Gamma \left| \frac{q_0}{e_0} \right| \) of their credit position \( \left| \frac{q_0}{e_0} \right| \), where \( \Gamma = \gamma \text{var}(e_1)^{\alpha} \), with \( \gamma \geq 0, \alpha \geq 0 \). The Appendix provides further details and regularity conditions for this constraint. As will become clear, our functional assumption regarding the diversion of funds is not only a convenient specification for tractability but also stresses the idea that financiers’ outside options increase in the size and volatility, or complexity, of their balance sheet. This constraint captures the relevant market practice in financial institutions whereby risk taking is limited not only by the overall size of the positions, position limits, but also by their expected

12. Given that the balance sheet consists of \( q_0 \) dollars and \( -\frac{q_0}{e_0} \) yen, the yen value of the financier’s liabilities is always equal to \( \left| \frac{q_0}{e_0} \right| \), irrespective of whether \( q_0 \) is positive or negative; hence the use of absolute value in the text. More formally, the financier’s creditors can recover a yen value equal to \( \max\left(1 - \Gamma \left| \frac{q_0}{e_0} \right|, 0 \right) \left| \frac{q_0}{e_0} \right| \).
riskiness, often measured by their variance. It is outside the scope of this article to provide deeper foundations for this constraint. The reader can think of it as a convenient specification of a more complicated contracting problem.\textsuperscript{13} Since creditors, when lending to the financier, correctly anticipate the incentives of the financier to divert funds, the financier is subject to a credit constraint of the form:

\[
\frac{V_0}{e_0} \geq \left| \frac{q_0}{e_0} \right| \quad \text{Intermediary Value in yen} \quad \text{Total Claims} \quad \text{Diverted Portion} \quad \text{Total divertable Funds}
\]

Limited commitment constraints in a similar spirit have been popular in the literature; for earlier use as well as foundations, see among others Caballero and Krishnamurthy (2001); Kiyotaki and Moore (1997); Hart and Moore (1994), and Hart (1995). Here we follow most closely the formulation in Gertler and Kiyotaki (2010) and Maggiori (2014).\textsuperscript{14}

The constrained optimization problem of the financier is:

\[
\begin{align*}
\max_{q_0} V_0 & = \mathbb{E} \left[ \beta \left( R - R^* e_1 \right) e_0 \right] q_0, & \text{subject to} & V_0 & \geq \Gamma \frac{q_0^2}{e_0}.
\end{align*}
\]

Since the value of the financier’s firm is linear in the position $q_0$, while the right-hand side of the constraint is convex in $q_0$, the constraint always binds.\textsuperscript{15} Substituting the firm’s value into the constraint and rearranging (using $R = \frac{1}{p}$), we

\begin{itemize}
  \item Such foundations could potentially be achieved in models of financial complexity where bigger and riskier balance sheets lead to more complex positions. In turn, these more complex positions are more difficult and costly for creditors to unwind when recovering their funds in case of a financier’s default.
  \item We generalize these constraints by studying cases where the outside option is directly increasing in the size of the balance sheet and its variance. Adrian and Shin (2014) provide foundations and empirical evidence for a value-at-risk constraint that shares some of the properties of our constraint.
  \item Intuitively, given any nonzero expected excess return in the currency market, the financier will want to either borrow or lend as much as possible in dollar and yen bonds. The constraint limits the maximum position and therefore binds. We make the very mild assumption that the model parameters always imply $|\gamma_0| \leq 1$. That is, we assume that the expected excess returns from currency speculation never exceed 100 percent in absolute value. This bound is several orders of magnitude greater than the expected returns in the data (of the order of 0–6 percent) and has no economic bearing on our model.
\end{itemize}
find: \( q_0 = \frac{1}{\Gamma} \mathbb{E}[e_0 - e_1 \frac{R^*}{R}] \). Integrating the above demand function over the unit mass of financiers yields the aggregate financiers’ demand for assets: \( Q_0 = \frac{1}{\Gamma} \mathbb{E}[e_0 - e_1 \frac{R^*}{R}] \). We collect this result in the lemma below.

**Lemma 2.** (Financiers’ downward sloping demand for dollars).

The financiers’ constrained optimization problem implies that the aggregate financial sector’s optimal demand for Dollar bonds versus Yen bonds follows:

\[
Q_0 = \frac{1}{\Gamma} \mathbb{E}\left[ e_0 - e_1 \frac{R^*}{R} \right],
\]

where

\[
\Gamma = \gamma (\text{var}(e_1))^{\alpha}.
\]

The demand for dollars decreases in the strength of the dollar (i.e., increases in \( e_0 \)), controlling for the future value of the dollar (i.e., controlling for \( e_1 \)). Notice that \( \Gamma \) governs the ability of financiers to bear risks; hence in the rest of the article we refer to \( \Gamma \) as the financiers’ risk-bearing capacity. The higher \( \Gamma \), the lower the financiers’ risk-bearing capacity, the steeper their demand curve, and the more segmented the asset market. To understand the behavior of this demand, let us consider two polar opposite cases. When \( \Gamma = 0 \), financiers are able to absorb any imbalances, that is, they want to take infinite positions whenever there is a nonzero expected excess return in currency markets. So uncovered interest rate parity (UIP) holds: \( \mathbb{E}[e_0 - e_1 \frac{R^*}{R}] = 0 \). When \( \Gamma \uparrow \infty \), then \( Q_0 = 0 \); financiers are unwilling to absorb any imbalances, that is, they do not want to take any positions, no matter what the expected returns from risk taking. In the intermediate cases (\( 0 < \Gamma < \infty \)) the model endogenously generates a deviation from UIP and relates it to financiers’ risk taking. On the contrary, since the covered interest rate parity (CIP) condition is an arbitrage involving no risk, it is always satisfied. Similarly the model smoothly converges to the frictionless benchmark (\( \Gamma \downarrow 0 \)) as the economy becomes deterministic (\( \text{var}(e_1) \downarrow 0 \)). Section III.A studies the carry trade and provides further analysis on UIP and CIP.

Since \( \Gamma \), the financiers’ risk-bearing capacity, plays a crucial role in our theory, we refer hereafter to the setup described so far as the basic gamma model. In many instances, like the one above, it is most intuitive to consider comparative statics on \( \Gamma \) rather
than its subcomponents, and we do so for the remainder of the article; in some instances it is interesting to consider the effect of each subcomponent $\gamma$ and $\text{var}(e_1)$ separately.\textsuperscript{16}

We stress that the demand function captures the spirit of international financial intermediation by providing a simple and tractable specification for the constrained portfolio problem that generates the demand function that has been central to the limits of arbitrage theory pioneered by De Long et al. (1990a,b), Shleifer and Vishny (1997), and Gromb and Vayanos (2002). We follow the pragmatic tradition of macroeconomics and frictional finance, and we take as given the prevalence of frictions and short-term debt in different currencies and proceed to analyze their equilibrium implications. This direct approach to modeling financial imperfections has a long-standing tradition and has proved very fruitful with recent contributions by Kiyotaki and Moore (1997); Gromb and Vayanos (2002); Mendoza, Quadrichi, and Ríos-Rull (2009); Mendoza (2010); Gertler and Kiyotaki (2010); Gârleanu and Pedersen (2011); Perri and Quadrini (2014).\textsuperscript{17}

For simplicity, we assume (for now and for much of this article) that financiers rebate their profits and losses to the Japanese households, not the U.S. ones. This asymmetry gives much tractability to the model, at fairly little cost to the economics.\textsuperscript{18}

Before moving to the equilibrium, note that we are modeling the ability of financiers to bear substantial risks over a horizon that ranges from a quarter to a few years. Our model is silent on the high-frequency market-making activities of currency desks in investment banks. To make this distinction intuitive, let us consider that the typical daily volume of foreign exchange transactions is estimated to be $5.3$ trillion.\textsuperscript{19} This trading is highly

\textsuperscript{16}The reader is encouraged either to intuitively consider the case $\alpha = 0$, or to follow the formal proofs that show the sign of the comparative statics to be invariant in $\Gamma$ and $\gamma$.

\textsuperscript{17}Even in the most recent macrofinance literature in a closed economy, intense foundations of the contracting environment have either been excluded or relegated to separate companion pieces (Brunnermeier and Sannikov 2014; He and Krishnamurthy 2013). See Duffie (2010) for an overview.

\textsuperscript{18}For completeness, note that this assumption had already been implicitly made in deriving the U.S. households' intertemporal budget constraint in equation (4). This assumption is relaxed in the Online Appendix, where we solve for general and symmetric payoff functions numerically.

\textsuperscript{19}Source: Bank of International Settlements (2013).
concentrated among the market-making desks of banks and is the subject of attention in the market microstructure literature pioneered by Evans and Lyons (2002). Although these microstructure effects are interesting, we abstract away from these activities by assuming that there is instantaneous and perfect risk sharing across financiers, so that any trade that matches is executed frictionlessly and nets out. We are only concerned with the ultimate risk, most certainly a small fraction of the total trading volume, which financiers have to bear over quarters and years because households’ demand is unbalanced.²⁰

II.C. Equilibrium Exchange Rate

Recall that for simplicity we are for now only considering imbalances resulting from trade flows (imbalances from portfolio flows will soon follow). The key equations of the model are the financiers’ demand:

\[ Q_0 = \frac{1}{1 + \frac{R}{R^*}} \left[ e_0 - e_1 \frac{R}{R^*} \right] , \]  

(12)

and the equilibrium “flow” demand for dollars in the dollar-yen market at times \( t = 0,1 \):

\[ \xi_0 e_0 - \iota_0 + Q_0 = 0, \]  

(13)

\[ \xi_1 e_1 - \iota_1 - RQ_0 = 0. \]  

(14)

Equation (13) is the market clearing equation for the dollar against yen market at time 0. It states that the net demand for dollar against yen has to be 0 for the market to clear. The net demand has two components: \( \xi_0 e_0 - \iota_0 \), from U.S. net exports, and \( Q_0 \), from financiers. Recall that we assume that U.S. households do not hold any currency exposure: they convert their Japanese sales of \( \xi_0 \) yen into dollars, for a demand

²⁰. This is consistent with evidence that market-making desks in large investment banks, for example Goldman Sachs, might intermediate very large volumes on a daily basis but are almost always carrying no residual risk at the end of the business day. In contrast, proprietary trading desks (before recent changes in legislation) or investment management divisions of the same investment banks carry substantial amounts of risk over horizons ranging from a quarter to a few years. These investment activities are the focus of this article. Similarly, our financiers capture the risk-taking activities of hedge funds and investment managers that have no market making interests and are therefore not the center of attention in the microstructure literature.
\[ \xi_0 e_0 \] of dollars. Likewise, Japanese households have \( t_0 \) dollars’ worth of exports to the United States and sell them, as they only keep yen balances.\(^{21}\) At time 1, equation (14) shows that the same net-export channel generates a demand for dollars of \( \xi_1 e_1 - t_1 \), whereas the financiers need to sell their dollar position \( RQ_0 \) that has accrued interest at rate \( R \).\(^{22}\) We now explore the equilibrium exchange rate in this simple setup.

1. Equilibrium Exchange Rate: A First Pass. To streamline the algebra and concentrate on the key economic content, we assume for now that \( \beta = \beta^* = 1 \), which implies \( R = R^* = 1 \), and that \( \xi_t = 1 \) for \( t = 0, 1 \). Adding equations (13) and (14) yields the U.S. external intertemporal budget constraint:

\[ e_1 + e_0 = t_0 + t_1. \]

Taking expectations on both sides: \( E[e_1] = t_0 + E[t_1] - e_0 \). From the financiers’ demand equation we have:

\[ E[e_1] = e_0 - \Gamma Q_0 = e_0 - \Gamma(t_0 - e_0) = (1 + \Gamma)e_0 - \Gamma t_0, \]

where the second equality follows from equation (13). Equating the two expressions for the time-one expected exchange rate, we have:

\[ E[e_1] = t_0 + E[t_1] - e_0 = (1 + \Gamma)e_0 - \Gamma t_0. \]

Solving this linear equation for the exchange rate at time 0, we conclude:

\[ e_0 = \frac{(1 + \Gamma)t_0 + E[t_1]}{2 + \Gamma}. \]

We define \( \{X\} \equiv X - E[X] \) to be the innovation to a random variable \( X \). Then, the exchange rate at time \( t = 1 \) is:

\[ e_1 = t_0 + t_1 - e_0 = t_0 + E[t_1] + \{t_1\} - e_0 \]
\[ = \{t_1\} + t_0 + E[t_1] - \frac{(1 + \Gamma)t_0 + E[t_1]}{2 + \Gamma} = \{t_1\} + \frac{t_0 + (1 + \Gamma)E[t_1]}{2 + \Gamma}. \]

21. These assumptions are relaxed in Section II.D and in the Online Appendix where households are allowed to have (limited) foreign currency positions.

22. At the end of period 0, the financiers own \( Q_0 \) dollars and \( \frac{Q_0}{e_0} \) yen. Therefore, at the beginning of period 1, they hold \( RQ_0 \) dollars and \( \frac{RQ_0}{e_0} \) yen. At time 1, they unwind their positions and give the net profits to their principals, which we assume for simplicity to be the Japanese households. Hence they sell \( RQ_0 \) dollars in the dollar-yen market at time 1.
This implies that \( \text{var}(e_1) = \text{var}(\xi_1) \), so that by equation (11), \( \Gamma = \gamma \text{var}(\xi_1) \).

We collect these results in the proposition below.

**PROPOSITION 1.** (Basic gamma equilibrium exchange rate). Assume that \( \xi_t = 1 \) for \( t=0,1 \), and that interest rates are zero in both countries. The exchange rate follows:

\[
e_0 = \frac{(1 + \Gamma)\xi_0 + \mathbb{E}[\xi_1]}{2 + \Gamma},
\]

\[
e_1 = \{\xi_1\} + \frac{\xi_0 + (1 + \Gamma)\mathbb{E}[\xi_1]}{2 + \Gamma},
\]

where \( \{\xi_1\} \) is the time 1 import shock. The expected dollar appreciation is:

\[
\mathbb{E}[e_0 - e_1] = \frac{\Gamma(\xi_0 - \mathbb{E}[\xi_1])}{(1 + \Gamma)\xi_0 + \mathbb{E}[\xi_1]}. 
\]

Furthermore, \( \Gamma = \gamma \text{var}(\xi_1) \).

Depending on \( \Gamma \), the time 0 exchange rate varies between two polar opposites: the UIP-based and the financial-autarky exchange rates. Both extremes are important benchmarks of open economy analysis, and the choice of \( \Gamma \) allows us to modulate the model between these two useful benchmarks. \( \Gamma \uparrow \infty \) results in \( e_0 = \frac{\xi_0 - \mathbb{E}[\xi_1]}{\xi_0} \), which we showed in Section II.A to be the financial autarky value of the exchange rate. Intuitively, financiers have so little risk-bearing capacity that no financial flows can occur between countries, and therefore trade has to be balanced period by period. When \( \Gamma = 0 \), UIP holds and we obtain \( e_0 = \frac{\xi_0 + \mathbb{E}[\xi_1]}{2} \). Intuitively, financiers are so relaxed about risk taking that they are willing to take infinite positions in currencies whenever there is a positive expected excess return from doing so. UIP only imposes a constant exchange rate in expectation \( \mathbb{E}[e_1] = e_0 \); the level of the exchange rate is then obtained by additionally using the intertemporal budget constraint in equation (15).

To further understand the effect of \( \Gamma \), notice that at the end of period 0 (say, time 0\(^+\)), the U.S. net foreign asset (NFA) position is \( N_0 = \xi_0 e_0 - \xi_0 = \frac{\mathbb{E}[\xi_1] - \xi_0}{2 + \Gamma} \). Therefore, the United States has positive NFA at \( t = 0^+ \) iff \( \xi_0 < \mathbb{E}[\xi_1] \). If the United States has a positive NFA position, then financiers are long the yen and short the dollar. For financiers to bear this risk, they require a compensation: the yen needs to appreciate in expectation. The required appreciation is generated by making the yen weaker at time 0. The magnitude of the effect depends on the extent of the financiers’ risk-bearing
capacity ($\Gamma$), as formally shown by taking partial derivatives:
\[
\frac{\partial e}{\partial t} = \frac{\partial}{\partial t} \left[ \frac{e(t) - E(t)}{2 + \Gamma} \right] = \frac{-N_{0^+}}{2 + \Gamma}.
\]
We collect the result in the proposition below.

**PROPOSITION 2.** (Effect of financial disruptions on the exchange rate). In the basic gamma model, we have
\[
y \frac{de_0}{d\gamma} = \frac{\partial}{\partial \gamma} \left[ \frac{e_0}{N_{0^+} + 2 + \frac{1}{2 + \Gamma}} \right] = \frac{-N_{0^+}}{2 + \Gamma},
\]
where $N_{0^+} = \frac{E(t) - e_0}{2 + \Gamma}$ is the U.S. net foreign asset (NFA) position. When there is a financial disruption ($\uparrow \gamma$, $\uparrow \Gamma$), countries that are net external debtors ($N_{0^+} < 0$) experience a currency depreciation ($\uparrow e$), while the opposite is true for net creditor countries.

Intuitively, net external debtor countries have borrowed from the world financial system, thus generating a long exposure for financiers to their currencies. Should the financial system’s risk-bearing capacity be disrupted, these currencies would depreciate to compensate financiers for the increased (perceived) risk. This modeling formalizes a number of external crises where broadly defined global risk aversion shocks, embodied here in $\Gamma$, caused large depreciations of the currencies of countries that had recently experienced large capital inflows. Della Corte, Riddiough, and Sarno (2014) confirm our theoretical prediction in the data. They show that net debtor countries’ currencies have higher returns than net creditors’ currencies, tend to be on the receiving end of carry trade-related speculative flows, and depreciate when financial disruptions occur. In this basic model the entire external balance of a country is absorbed by the financier; we relax this shortly by providing a distinct role between $Q$ and the external balance. Here we clarify that the driving force behind the result in Proposition 2 is the position of the financiers, that is, what matters for the effect of an increase in $\Gamma$ on the exchange rate is whether $Q$ is positive or negative. The proposition stresses the idea that financiers are more likely to be long the currency of debtor countries since these countries have borrowed from the world financial system.

To illustrate how the results derived so far readily extend to more general cases, we report expressions allowing for stochastic U.S. export shocks $\xi_t$, as well as nonzero interest rates. Several

23. In mapping the proposition into the data, one can think that the net foreign asset positions are correlated with $Q$, but the correlation can be less than perfect, with instances like the United States, where the two might be substantially different (see Shin 2012; Maggiori 2014).
more extensions can be found in Section IV and the Online Appendix.

PROPOSITION 3. With general trade shocks and interest rates \((\xi_t, R_t, R^*)\), the values of exchange rate at times \(t = 0, 1\) are:

\[
e_0 = \frac{E\left[\frac{\xi_0 + \xi_1}{\xi_1}\right] + \frac{\Gamma_0}{R^*}}{E\left[\frac{\xi_0 + \xi_1}{\xi_1}\right] + \frac{\Gamma_0}{R^*}}; \quad e_1 = E[e_1] + \{e_1\},
\]

where we again denote by \(\{X\} = X - E[X]\) the innovation to a random variable \(X\), and

\[
E[e_1] = \frac{R}{R^*} \frac{E\left[R^* \left(\xi_0 + \frac{\xi_1}{\xi_1}\right)\right]}{E\left[R^* \left(\frac{\xi_0}{\xi_1} + \frac{\xi_1}{\xi_1}\right)\right]} + \Gamma \xi_0,
\]

\[
\{e_1\} = \left\{\frac{\xi_1}{\xi_1}\right\} + R \frac{E\left[R^* \left(\frac{\xi_0}{\xi_1} + \frac{\xi_1}{\xi_1}\right)\right]}{E\left[R^* \left(\frac{\xi_0}{\xi_1} + \frac{\xi_1}{\xi_1}\right)\right]} + \Gamma \xi_0 \left\{\frac{1}{\xi_1}\right\}.
\]

When \(\xi_1\) is deterministic, \(\Gamma = \text{var}(\frac{\xi_1}{\xi_1})\). The proof of this proposition reports the corresponding solution for \(\Gamma\) when \(\xi_1\) is stochastic.

II.D. The Impact of Portfolio Flows

We now further illustrate how the supply and demand of assets do matter for the financial determination of the exchange rate. We stress the importance of portfolio flows in addition and perhaps more important than trade flows for our framework. The basic model so far has focused on current account– or net foreign asset–based flows; we now introduce pure portfolio flows that alter the countries’ gross external positions. We focus on the simplest form of portfolio flows from households, not so much for their complete realism but because they allow for the sharpest analysis of the main forces of the model. The Online Appendix extends this minimalistic section to more general flows.

1. Asset Flows Matter in the Gamma Model. Consider the case where Japanese households have, at time 0, an inelastic demand (e.g., some noise trading) \(f^*\) of dollar bonds funded by an
offsetting position $-\frac{f^*}{e_0}$ in yen bonds. Both transactions face the financiers as counterparties.

Although we take these flows as exogenous, they can be motivated as a liquidity shock, or perhaps as a decision resulting from bounded rationality or portfolio delegation. Technically, the maximization problem for the Japanese household is the one written before, where the portfolio flow is not a decision variable coming from maximization but simply an exogenous action. 24

The flow equations are now given by:

\[(20)\quad \xi_0 e_0 - \xi_1 t_0 + Q_0 + f^* = 0, \quad \xi_1 e_1 - t_1 - RQ_0 - Rf^* = 0.\]

The financiers’ demand is still $Q_0 = \frac{1}{f} e_0 - R e_1$. The equilibrium exchange rate is derived in the proposition below.

**Proposition 4.** (Gross capital flows and exchange rates). Assume $t = R = R^* = 1$ for $t = 0,1$. With an inelastic time 0 additional demand $f^*$ for dollar bonds by Japanese households who correspondingly sell $-\frac{f^*}{e_0}$ of yen bonds, the exchange rates at times $t = 0,1$ are:

\[e_0 = \frac{(1 + \Gamma)t_0 + \mathbb{E}[t_1] - \Gamma f^*}{2 + \Gamma}; \quad \xi_1 e_1 - t_1 = \frac{t_0 + (1 + \Gamma)\mathbb{E}[t_1] + \Gamma f^*}{2 + \Gamma}.\]

Hence, additional demand $f^*$ for dollars at time zero induces a dollar appreciation at time 0 and subsequent depreciation at time 1. However, the time-average value of the dollar is unchanged: $e_0 + e_1 = t_0 + t_1$, independently of $f^*$.

Furthermore, $\gamma = \gamma var(t_1)^\gamma$.

**Proof.** Define: $\tilde{t}_0 = \tilde{t}_0 - f^*$, and $\tilde{t}_1 = \tilde{t}_1 + f^*$. Given equations (20), our “tilde” economy is isomorphic to the basic economy considered in equations (13) and (14). For instance, import demands are now $\tilde{t}_t$ rather than $t_t$. Hence, Proposition 1 applies to this tilde economy, thus implying that:

\[e_0 = \frac{(1 + \Gamma)\tilde{t}_0 + \mathbb{E}[\tilde{t}_1] - \Gamma f^*}{2 + \Gamma}, \quad \xi_1 e_1 - \tilde{t}_1 = \frac{t_0 + (1 + \Gamma)\mathbb{E}[\tilde{t}_1] + \Gamma f^*}{2 + \Gamma}.\]

24. The Japanese households’ state-by-state budget constraint is $\sum_{t=0}^{1} r_{\gamma t}^t + p_t^f r_{\gamma t}^t = \sum_{t=0}^{1} C_t = \sum_{t=0}^{1} (\xi_t + Q_0 - R e_1 - Q_{t+1}),$ where $\pi_t$ are FX trading profits of the Japanese, so, $\pi_0 = 0$, $\pi_1 = \frac{(\xi + Q_0 - R e_1 - Q_{t+1})}{e_1}$ (recall that the financiers rebate their profits to the Japanese).
An increase in Japanese demand for dollar bonds needs to be absorbed by financiers, who correspondingly need to sell dollar bonds and buy yen bonds. To induce financiers to provide the desired bonds, the dollar needs to appreciate on impact as a result of the capital flow to then be expected to depreciate, thus generating an expected gain for the financiers' short dollar positions. This example emphasizes that our model is an elementary one in which a relative price, the exchange rate, has to move to equate the supply and demand of two assets, yen and dollar bonds. The capital flows considered in this section are gross flows that do not alter the net foreign asset position, thus introducing a first example of the distinct role for the financiers’ balance sheet from the country net foreign asset position. In the data gross flows are much larger than net flows, and we provide a reason they play an important role in determining the exchange rate.25

This framework can analyze concrete situations, such as the recent large-scale capital flows from developed countries into emerging market local-currency bond markets, say, by U.S. investors into Brazilian real bonds, that put upward pressure on the receiving countries’ currencies. While such flows and their effect on currencies have been paramount in the logic of market participants and policy makers, they had thus far proven elusive in formal theoretical analysis.

Hau, Massa, and Peress (2010) provide direct evidence that plausibly exogenous capital flows affect the exchange rate in a manner consistent with the gamma model. They show that following a restating of the weights of the MSCI World Equity Index, countries that as a result experienced capital inflows (because their weight in the index increased) saw their currencies appreciate.

To stress the difference between our basic gamma model of the financial determination of exchange rates in imperfect financial markets and the traditional macroeconomic framework, we illustrate two polar cases that have been popular in previous literature: the UIP-based exchange rate and the complete market exchange rate.26

25. One could extend the distinction between country-level positions and financiers’ balance sheet further by modeling situations where not all gross flows are stuck, either temporarily or permanently, on the balance sheet of the financiers.

26. For models of portfolio ows see also Froot et al. (2001), Froot and Stein (1991), and Ivanshina et al. (2012).
i. Financial Flows in a UIP Model. Much of the now classic international macroeconomic analysis spurred by Dornbusch (1976) and Obstfeld and Rogoff (1995) either directly assumes that UIP holds or effectively imposes it by solving a first-order linearization of the model.\(^{27}\) The closest analog to this literature in the basic gamma model is the case where \(\Gamma = 0\), such that UIP holds by assumption. In this world, financiers are so relaxed, that is, their risk-bearing capacity is so ample, about supplying liquidity to satisfy shifts in the world demand for assets that such shifts have no impact on expected returns. Consider the example of U.S. investors suddenly wanting to buy Brazilian real bonds; in this case financiers would simply take the other side of the investors’ portfolio demand with no effect on the exchange rate between the dollar and the real. In fact, Proposition 4 confirms that if \(\Gamma = 0\), then portfolio flow \(f^*\) has no effect on the equilibrium exchange rate.\(^{28}\)

ii. Financial Flows in a Complete Market Model. Another strand of the literature has analyzed risk premia predominantly under complete markets. We now show that the exchange rate in a setup with complete markets (and no frictions) but otherwise identical to ours is constant, and therefore trivially not affected by the flows.

**Lemma 3.** (Complete Markets). In an economy identical to the setup of the basic gamma model, other than the fact that financial markets are complete and frictionless, the equilibrium exchange rate is constant: \(e_t = \nu\), where \(\nu\) is the relative Negishi weight of Japan.

Here, we only sketch the logic and the main equations; a full treatment is relegated to the Online Appendix. Under complete markets, the marginal utility of U.S. and Japanese agents must be equal when expressed in a common currency. Intuitively, the full risk sharing that occurs under complete markets calls for Japan and the United States to have the same marginal benefit from consuming an extra unit of nontradables. In our setup, this

\(^{27}\) Intuitively, a first-order linearization imposes certainty equivalence on the model and therefore kills any risk premia such as those that could generate a deviation from UIP.

\(^{28}\) These gross flows do not play a role in determining the exchange rate even in models, for example Schmitt-Grohé and Uribe (2003), that assume reduced-form deviations from UIP to be convex functions of the net foreign asset position.
risk-sharing condition takes a simple form: $e_t = \nu$, where \( \nu \) is a constant.\(^{29}\) Simple substitution of the conditions $C_{NT,t} = \chi_t$ and $C^*_{NT,t} = \chi^*_t$ shows that $e_t = \nu$, that is, the exchange rate is constant.\(^{30}\)

2. Flows, Not Just Stocks, Matter in the Gamma Model. In frictionless models only stocks matter, not flows per se. In the gamma model, instead, flows per se matter. This is a distinctive feature of our model. To illustrate this, consider the case where the United States has an exogenous dollar-denominated debt toward Japan, equal to $D_0$ due at time 0, and $D_1$ due at time 1.\(^{31}\) For simplicity, assume $\beta = \beta^* = R = R^* = \xi_t = 1$ for $t = 0,1$. Hence, total debt is $D_0 + D_1$. The flow equations now are:

$$e_0 - \iota_0 - D_0 + Q_0 = 0; \quad e_1 - \iota_1 - D_1 - Q_0 = 0.$$  

The exchange rate at time 0 is:

$$e_0 = \frac{(1 + \Gamma)\iota_0 + E[\iota_1]}{2 + \Gamma} + \frac{(1 + \Gamma)D_0 + D_1}{2 + \Gamma}.$$  

Hence, when finance is imperfect ($\Gamma > 0$), both the timing of debt flows, as indicated by the term $(1 + \Gamma)D_0 + D_1$, and the total stock of debt ($D_0 + D_1$) matter in determining exchange rates. The early flow, $D_0$, receives a higher weight $\left(\frac{1 + \Gamma}{2 + \Gamma}\right)$ than the late flow, $D_1$, $\left(\frac{1}{2 + \Gamma}\right)$. In sum, flows, not just stocks, matter for exchange rate determination.

To highlight the contrast, let us parameterize the debt repayments as $D_0 = F$ and $D_1 = -F + S$. The parameter $F$ alters the

---

29. Formally, the constant is the relative Pareto weight assigned to Japan in the planner’s problem that solves for complete-market allocations.

30. The irrelevance of the $f$ gross flows generalizes also to complete and incomplete market models where the exchange rate is not constant and the presence of a risk premium makes the two currencies imperfect substitutes. Intuitively in these models the state variables are ratios of stocks of assets, such as wealth, and since these gross flows do not alter the value of such stocks, they have no equilibrium effects because the agents can frictionlessly unwind them. In our model they have effects because these flows alter the balance sheet of constrained financiers.

31. Hence, the new budget constraint is $\sum_{t=0}^{1} R^{-t}(Y_{NT,t} + p_{H,t}Y_{H,t} - D_t) = \sum_{t=0}^{1} R^{-t}(C_{NT,t} + p_{H,t}C_{H,t} + p_{F,t}C_{F,t})$. 
flow of debt repayment at time 0 but leaves the total stock of debt \((D_0 + D_1 = S)\) unchanged. The parameter \(S\) instead alters the total stock of debt, but does not affect the flow of repayment at time 0. We note that \(\frac{de_0}{dF} = \frac{\Gamma}{2 + \Gamma}\) and \(\frac{de_0}{dS} = \frac{1}{2 + \Gamma}\). When \(\Gamma \uparrow \infty\), only flows affect the exchange rate at time 0; this is so even when flows leave the total stocks unchanged \((\frac{de_0}{dF} > 0 = \frac{de_0}{dS})\). In contrast, when finance is frictionless \((\Gamma = 0)\), flows have no effect on the exchange rate, and only stocks matter \((\frac{de_0}{dF} = 0 < \frac{de_0}{dS})\). We collect the result in the proposition below.

**Proposition 5.** (Stock versus Flow Matters in the Gamma Model). Flows matter for the exchange rate when \(\Gamma > 0\). In the limit when financiers have no risk-bearing capacity \((\Gamma \uparrow \infty)\), only flows matter. When risk-bearing capacity is very ample \((\Gamma = 0)\), only stocks matter.

3. The Exchange Rate Disconnect. The Meese and Rogoff (1983) result on the inability of economic fundamentals such as output, inflation, exports, and imports to predict or even contemporaneously comove with exchange rates has had a chilling and long-lasting effect on theoretical research in the field (see Obstfeld and Rogoff 2001).\(^{32}\) The gamma model helps reconcile the disconnect by introducing financial forces, both the risk-bearing capacity \(\Gamma\) and the balance sheet \(Q\), as determinants of exchange rates. Intuitively a disconnect occurs because economies with identical fundamentals feature different equilibrium exchange rates depending on the incentives of the financiers to hold the resulting (gross) global imbalances.

Recently new evidence has been building in favor of these new financial channels. In addition to the instrumental variables approach in Hau, Massa, and Peress (2010) discussed earlier, Froot and Ramadorai (2005), Adrian, Etula, and Shin (2009), Adrian, Etula, and Groen (2011), Hong and Yogo (2012), and Kim, Liao, and Tornell (2014) find that flows, financial conditions, and financiers’ positions provide information about expected currency returns. Froot and Ramadorai (2005) show that

\(^{32}\) Some forecastability of exchange rates using traditional fundamentals appears to occur at very long horizons (e.g., 10 years) in Mark (1995) or for specific currencies, such as the U.S. dollar, using transformations of the balance of payments data (Gourinchas and Rey 2007b; Gourinchas, Govillot, and Rey 2010).
medium-term variation in expected currency returns is mostly associated with capital flows, whereas long-term variation is more strongly associated with macroeconomic fundamentals. Hong and Yogo (2012) show that speculators’ positions in the futures currency market contain information that is useful, beyond the interest rate differential, to forecast future currency returns. Adrian, Etula, and Groen (2011) and Adrian, Etula, and Shin (2009) show that empirical proxies for financial conditions and the tightness of financiers’ constraints help forecast both currency returns and exchange rates. Kim, Liao, and Tornell (2014) show that information extracted from the speculators’ positions in the futures currency market helps predict exchange rate changes at horizons between 6 and 12 months.

The model can also help rationalize the comovement across bilateral exchange rates and between exchange rates and other asset classes. Intuitively, this occurs because all these assets are traded by financiers and are therefore affected to some degree by the same financial forces. Verdelhan (2013) shows that there is substantial comovement between bilateral exchange rates both in developed and emerging economies, while Dumas and Solnik (1995), Hau and Rey (2006), Verdelhan (2013), Farhi and Gabaix (2014), and Lettau, Maggiori, and Weber (2014) link movements in exchange rates to movements in equity markets.

II.E. Closing the Economy: Endowments, Production, and Unemployment

Very little has been said so far about output; we now close the model by describing the output market. To build the intuition for our framework, we consider first a full endowment economy and then production economies under both flexible and sticky prices.

1. Endowment Economy. Let all output stochastic processes \( Y_{NT,t}, Y_{H,t}, Y_{NT,t}, Y_{F,t} \) be exogenous strictly positive endowments. Assuming that all prices are flexible and that the law of one price (LOP) holds, one has \( p_{H,t} = p_{H,t}^* e_t \), and \( p_{F,t} = p_{F,t}^* e_t \).

Summing U.S. and Japanese demand for U.S. tradable goods \( C_{H,t} = \alpha_{H,t} \) and \( C_{H,t} = \frac{\delta_{H,t}}{p_{H,t}} \), respectively, which are derived as in Section II.A), we obtain the world demand for U.S. tradables:
\[
D_{H,t} \equiv C_{H,t} + C_{H,t} = \frac{\alpha_{H,t} + \delta_{H,t}}{p_{H,t}}.
\]
Clearing the goods market, \( Y_{H,t} = D_{H,t} \), yields the equilibrium price in dollars of U.S.
tradables: \( p_{H,t} = \frac{a_t + \xi_t e_t}{Y_{H,t}} \). An entirely similar argument yields:
\[
p_{F,t}^* = \frac{a_t + \xi_t}{Y_{F,t}}.
\]

2. Production without Price Rigidities. Let us introduce a minimal model of production that will allow us to formalize the effects of the exchange rate on output and employment. While we maintain the assumption that nontradable goods in each country are given by endowment processes, we now assume that tradable goods in each country are produced with a technology linear in labor with unit productivity. In each country, labor \( L \) is supplied inelastically and is internationally immobile.

Simple profit maximization at the firm level yields a dollar wage in the United States of \( w_{H,t} = p_{H,t} \). Under flexible prices, goods market clearing then implies full employment \( Y_{H,t} = L \) and a U.S. tradable price in dollars of \( p_{H,t}^* = \frac{a_t + \xi_t e_t}{L} \), where the circle in \( p^* \) denotes a frictionless quantity. Likewise, for Japanese tradables the equilibrium features both full employment \( Y_{F,t} = L \) and a yen price of \( p_{F,t}^{*0} = \frac{a_t + \xi_t}{L} \).

3. Production with Price Rigidities. Let us now assume that wages are “downward rigid” in domestic currency at a preset level of \( \bar{p}_H, \bar{p}_F \), where these prices are exogenous. Let us further assume that firms do not engage in pricing to market, so that prices are sticky in producer currency (PCP). Firm profit maximization then implies that: \( p_{H,t} = \max(\bar{p}_H, p_{H,t}^0) \); or more explicitly: \( p_{H,t} = \max(\bar{p}_H, \frac{a_t + \xi_t e_t}{L}) \). Hence
\[
Y_{H,t} = \min\left(\frac{a_t + e_t \xi_t}{\bar{p}_H}, L\right).
\]

If demand is sufficiently low \( (a_t + \xi_t e_t < \bar{p}_H L) \), then output is demand-determined (i.e., it depends directly on \( e_t, \xi_t \), and \( a_t \) and there is unemployment: \( L - Y_{H,t} > 0 \). Notice that in this case the exchange rate has an expenditure-switching effect: if the dollar depreciates \( (e_t \uparrow) \), unemployment falls and output expands in the United States. Intuitively, since U.S. tradables’ prices are sticky in dollars, these goods become cheap for Japanese consumers to buy when the dollar depreciates. In a
world that is demand constrained, this expansion in demand for U.S. tradable goods is met by expanding production, thus raising U.S. output and employment.

Clearly, a similar expression and mechanism apply to Japanese tradables:

$$Y_{F,t} = \min\left(\frac{a^*_t + \frac{\pi_t}{e_t}}{p^*_t}, L\right).$$

The expenditure switching role of exchange rates has been central to the Keynesian analysis of open macroeconomics of Dornbusch (1976) and Obstfeld and Rogoff (1995). In the gamma model, it is enriched by being the central channel for the transmission of financial forces affecting the exchange rate, such as the risk-bearing capacity and balance sheet of the financiers, into output and employment.

The financial determination of exchange rates has real consequences. Let us reconsider the earlier example of a sudden inflow of capital from U.S. investors into Brazilian real bonds. The exchange rate in this economy with production and sticky prices is still characterized by Proposition 4. As previously discussed, the capital inflow in Brazil causes the real to appreciate, and if the flow is sufficiently strong (\(f\) sufficiently high) or the financiers’ risk-bearing capacity sufficiently low (\(\Gamma\) sufficiently high), the appreciation (the increase in \(e_0\)) can be so strong as to make Brazilian goods uncompetitive on international markets; the corresponding fall in world demand for Brazilian output (\(\downarrow C_{H,0} = \frac{-\pi_0}{e_0 p_F}\)) causes an economic slump in Brazil with both falling output and increasing unemployment.

The main focus of our model is to disconnect the exchange rate from fundamentals by altering the structure of financial markets. Of course, part of the disconnect in practice also comes from frictions in the goods markets. These frictions can be analyzed in our model; we illustrate this by considering prices that are sticky in the export destination currency (LCP). To make the point sharp, assume that prices for U.S. tradable

33. When \(\alpha = 0, \frac{\partial e_0}{\partial f} = -\frac{\Gamma}{\Gamma + \alpha} < 0\). More generally, a sufficient condition for this effect is that \(\alpha\) is small.

34. Brazilian Finance Minister Guido Mantega complained, as reported in Fontevecchia (2011), that “We have to face the currency war without allowing our productive sector to suffer. If we allow [foreign] liquidity to [freely] enter [the economy], it will bring the Dutch Disease to the economy.”
goods are exogenously set at \( \{ \bar{p}_H, \bar{p}^*_H \} \) in dollars in the United States and in yen in Japan, respectively.

**Lemma 4. (LCP versus PCP),** Under LCP the value of the exchange rate is the same as under PCP, but US tradable output does not depend on the exchange rate: \( Y_{H,t} = \min \left( \frac{\alpha}{\bar{p}_H + \frac{\xi}{\bar{p}^*_H}, L} \right) \).

*Proof.* Because of the log specification, the dollar value of U.S. imports and exports is unchanged: they are still \( \iota_t \) and \( \epsilon_t \xi_t \). Consequently, the value of net exports is unchanged, and the exchange rate is unchanged from the previous formulas. Total demand is derived as before. \( \square \).

LCP helps further the disconnect between the exchange rate and fundamentals by preventing output in the tradable sector from responding to the exchange rate.35

### III. Revisiting Canonical Issues with the Gamma Model

We consider a number of canonical issues of international macroeconomics via the lenses of the gamma model. While these classic issues have also been the subject of previous literature, our analysis not only provides new insights but also allows us to illustrate how the framework built in the previous section provides a unified and tractable rationalization of empirical regularities that are at the center of open economy analysis.

#### III.A. The Carry Trade in the Presence of Financial Shocks

In the gamma model there is a profitable carry trade. Let us give the intuition in terms of the most basic model first and then extend it to a setup with shocks to the financiers’ risk-bearing capacity (\( \Gamma \) shocks).

35. Devereux and Engel (2003) stressed the absence of exchange rate effects on output under LCP. The empirical evidence shows that in practice, a combination of PCP, LCP, and limited pass-through are present in the data (see Gopinath and Itskhoki 2010; Gopinath, Itskhoki, and Rigobon 2010; Amiti, Itskhoki, and Konings 2014; Burstein and Gopinath 2015). For much of this article, we focus on flexible prices or PCP as the basic cases. As shown in Lemma 4, our qualitative analysis can easily accommodate a somewhat more limited pass-through of exchange rate changes to local prices of internationally traded goods.
First, imagine a world in which countries are in financial autarky because the financiers have zero risk-bearing capacity ($\Gamma = \infty$), suppose that Japan has a 1 percent interest rate while the United States has a 5 percent interest rate, and that all periods ($t = 0, ..., T$) are ex-ante identical with $\xi_t = 1$ and $\xi_t$ a martingale. Thus, we have $e_t = \xi_t$, and the exchange rate is a random walk $e_0 = \mathbb{E}[e_1] = ... = \mathbb{E}[e_T]$. A small financier with some available risk-bearing capacity, for example, a small hedge fund, could take advantage of this trading opportunity and pocket the 4 percent interest rate differential. In this case, there is a very profitable carry trade. As the financial sector risk-bearing capacity expands ($\Gamma$ becomes smaller, but still positive), this carry trade becomes less profitable, but does not disappear entirely unless $\Gamma = 0$, in which case the UIP condition holds. Intuitively, the carry trade in the basic gamma model reflects the risk compensation necessary to induce the financiers to intermediate global financial flows.

In the most basic model, the different interest rates arise from different rates of time preferences, such that $R = \beta^{-1}$ and $R^* = \beta^{-1}$. Without loss of generality, assume $R < R^*$ so that the dollar is the “funding” currency, and the yen the “investment” currency. The return of the carry trade is $R^c = \frac{R^* e_1}{R e_0} - 1$. For notational convenience we define the carry trade expected return as $\bar{R}^c = \mathbb{E}[R^c]$. The calculations in Proposition 3 allow us to immediately derive the equilibrium carry trade.

**Proposition 6.** Assume $\xi_t = 1$. The expected return to the carry trade in the basic gamma model is:

$$
\bar{R}^c = \Gamma \frac{R^* \mathbb{E}[e_1] - \xi_0}{(R^* + \Gamma)\xi_0 + \frac{R^*}{\Gamma} \mathbb{E}[e_1]}, \text{ where } \Gamma = \gamma \text{var}(\xi_1)^{\beta}.
$$

Hence the carry trade return is bigger (i) when the return differential $\frac{R^*}{\Gamma}$ is larger (ii) when the funding country is a net foreign creditor (iii) when finance is more imperfect (higher $\Gamma$).

To gain further intuition on the foregoing result, consider first the case where $\xi_0 = \mathbb{E}[e_1]$. The first-order approximation to $\bar{R}^c$ in the case of a small interest rate differential $R^* - R$ is $\bar{R}^c \approx \frac{\Gamma}{2 + \Gamma} (R^* - R)$. Notice that we have both $\frac{\partial R^c}{\partial R^c} > 0$ and
\(\frac{\partial R}{\partial (R^* - R)} > 0\), so that the profitability of the carry trade increases the more limited the risk-bearing capacity of the financiers and the larger the interest rate differential.\(^3\)\(_6\)

The effects of broadly defined “global risk aversion,” here proxied by \(\Gamma\), on the profitability of the carry trade have been central to the empirical analysis of, for example, Brunnermeier, Nagel, and Pedersen (2009), Lustig, Roussanov, and Verdelhan (2011), and Lettau, Maggiori, and Weber (2014). Here we have shown that the carry trade is more profitable the lower the risk-bearing capacity of the financiers; next we formally account for shocks to such capacity in the form of a stochastic \(\Gamma\).

In addition to a pure carry force due to the interest rate differential, our model features global imbalances as a separate risk factor in currency risk premia. The reader should recall Proposition 2 that showed how net external debtor countries’ currencies have a positive excess return and depreciate whenever risk-bearing capacity decreases (\(\uparrow \Gamma\)). This effect occurs even if both countries have the same interest rate, thus being theoretically separate from the pure carry trade. Della Corte, Riddiough, and Sarno (2014) test these theoretical predictions and find evidence of a global imbalance risk factor in currency excess returns.\(^3\)\(_7\)

1. The Exposure of the Carry Trade to Financial Disruptions.

We now expand on the risks of the carry trade by studying a three-period \((t = 0,1,2)\) model with stochastic shocks to the financiers’ risk-bearing capacity in the middle period. To keep the analysis streamlined, we take period 2 to be the long run. Intuitively, period 2 will be a long-run steady state where countries have zero net foreign assets and run a zero trade balance. This allows us to quickly focus on the short- to medium-run exchange rate dominated by financial forces and the long-run exchange rate completely anchored by fundamentals. We jump into

\(^3\)\(_6\). The first effect occurs because given an interest rate differential, expected returns to the carry trade have to increase whenever the risk-bearing capacity of the financiers goes down to induce them to intermediate financial flows. The second effect occurs because given a level of risk-bearing capacity for the financiers, an increase in the interest rate differential will not be offset one to one by the expected exchange rate change due to the risk premium.

\(^3\)\(_7\). Notice that we built the model so that financial forces have no effect on the interest rates and the exchange rate makes all the adjustment; although this sharpens the model, we could extend the framework to allow for effects of imbalances on both the exchange rate and interest rates.
the analysis and provide many of the background details of this model in the Online Appendix.\(^{38}\)

We assume that time 1 financial conditions, \(\Gamma_1\), are stochastic. In the three-period economy with a long-run last period, the equilibrium exchange rates are:

\[
e_0 = \frac{\Gamma_0 t_0 + \frac{R^*}{R} \mathbb{E}_0 \left[ \frac{\Gamma_{1t+1} + \frac{R^*}{R}}{\Gamma_{1t+1}} \right]}{\Gamma_0 + 1}; \quad e_1 = \frac{\Gamma_{1t1} + \frac{R^*}{R} \mathbb{E}_1 [t_2]}{\Gamma_1 + 1}; \quad e_2 = t_2.
\]

Recall that the carry trade return between period 0 and 1 is \(R^c \equiv \frac{R^* e_1}{R e_0} - 1\). Interestingly, in this case the carry trade also has "exposure to financial conditions." Notice that \(\frac{\partial e_1}{\partial t_0} < 0\) in the equations above, so that the dollar (the funding currency) appreciates whenever there is a negative shock to the financiers' risk-bearing capacity (\(\Gamma_1\)). Since in our chosen parameterization the carry trade is short dollar and long yen, we correspondingly have \(\frac{\partial R^c}{\partial t_0} < 0\), the carry trade does badly whenever there is a negative shock to the financiers' risk-bearing capacity (\(\Gamma_1\)). This is consistent with the intuition and the empirical findings in Brunnermeier, Nagel, and Pedersen (2009); we obtain this effect here in the context of an equilibrium model. We formalize and prove the results obtained so far in the proposition below.

**PROPOSITION 7. (Determinants of Expected Carry Trade Returns).**

Assume that \(R^* > R\), \(1 = t_0 = \mathbb{E}_0 [t_1]\), and \(t_1 = \mathbb{E}_1 [t_2]\). Define the "certainty equivalent" \(\tilde{\Gamma}_1\) by \(\tilde{\Gamma}_1 \equiv \mathbb{E}_0 \left[ \frac{\Gamma_{1t1} + \frac{R^*}{R}}{\Gamma_{1t1}} \right]\). Consider the returns to the carry trade, \(R^c\). The corresponding expected return \(\tilde{R}^c \equiv \mathbb{E}_0 [R^c]\) is

\[
\tilde{R}^c = (R^* - 1) \Gamma_0 \frac{\tilde{\Gamma}_1 + 1 + R^*}{\tilde{\Gamma}_1 (\Gamma_0 + R^*) + \Gamma_0 + (R^*)^2},
\]

with \(R^* \equiv \frac{R^*}{R}\). We have:

(i) An adverse shock to financiers affects the returns to carry trade negatively: \(\frac{\partial R^c}{\partial \Gamma_1} < 0\).

(ii) The carry trade has positive expected returns: \(\tilde{R}^c > 0\).

\(^{38}\) The flow demand equations in the yen/dollar market are: \(e_t - t_t + Q_t = 0\) for \(t=0,1\), and in the long-run period \(e_2 - t_2 = 0\), with the financiers' demand for dollars: \(Q_t = \frac{\mathbb{E}_t [e_{t+1}]}{e_t} \) with \(\Gamma_t = \gamma \text{var}_t (e_{t+1})\).
The expected return to the carry trade is higher the worse the financial conditions are at time 0 \( \frac{dR^c}{dt} > 0 \), the better the financial conditions are expected to be at time 1 \( \frac{dR^c}{dt} < 0 \), and the higher the interest rate differential \( \frac{dR^c}{dt} > 0, \frac{dR^c}{dt} < 0 \).

2. The Fama Regression. The classic UIP regression of Fama (1984) is in levels:\(^{39}\)

\[
\frac{e_1 - e_0}{e_0} = \alpha + \beta_{\text{UIP}}(R - R^e) + \epsilon_1.
\]

Under UIP, we would find \( \beta_{\text{UIP}} = 1 \). However, a long empirical literature finds \( \beta_{\text{UIP}} < 1 \), and sometimes even \( \beta_{\text{UIP}} < 0 \). The proposition below rationalizes these findings in the context of our model.

**Proposition 8.** (Fama Regression and Market Conditions). The coefficient of the Fama regression is

\[
\beta_{\text{UIP}} = \frac{1+\tilde{\Gamma}_1-\Gamma_0}{(1+\Gamma_0)(1+\tilde{\Gamma}_1)}.
\]

Therefore one has \( \beta_{\text{UIP}} < 1 \) whenever \( \Gamma_0 > 0 \). In addition, one has \( \beta_{\text{UIP}} < 0 \) if and only if \( \tilde{\Gamma}_1 + 1 < \Gamma_0 \), that is, if risk-bearing capacity is very low in period 0 compared to period 1.

Intuitively financial market imperfections always lead to \( \beta_{\text{UIP}} < 1 \) and very bad current market imperfections compared to expected future ones lead to \( \beta_{\text{UIP}} < 0 \). This occurs because any positive \( \Gamma \) leads to a positive risk premium on currencies that the financiers are long of and hence to a deviation from UIP \( \beta_{\text{UIP}} < 1 \). If, in addition, financial conditions are particularly worse today compared to tomorrow the risk premium is so big as to induce currencies that have temporarily high interest rates to appreciate on average \( \beta_{\text{UIP}} < 0 \).

The intuition for \( \beta_{\text{UIP}} < 1 \) is as follows. In the language of Fama (1984), when Japan has high interest rates, the risk premium on the yen is high. The reason is that the risk premium is not entirely eliminated by financiers, who have limited risk-

\(^{39}\) The regression is most commonly performed in its logarithmic approximation version, but the levels prove more convenient for our theoretical treatment without loss of economic content.
bearing capacity. In the limit where finance is eliminated ($\Gamma = \infty$), an interest rate of 1 percent translates one-for-one into a risk premium of 1 percent ($\beta_{\text{UIP}} = 0$). If riskiness (assuming $\alpha > 0$) or financial frictions go to 0, then $\beta_{\text{UIP}}$ goes to 1.\footnote{As riskiness ($\text{var}(e_1), \text{var}(e_2)$) goes to 0, $\Gamma_0$ and $\Gamma_1$ go to 0, so $\beta_{\text{UIP}}$ goes to 1.} In all cases, covered interest rate parity (CIP) holds in the model. This is because we allow financiers to eliminate all riskless arbitrages. Online Appendix Section A.3.C provides full details on arbitrage trading in our model. There, we formulate a version of our basic demand (equation (10)), that applies to an arbitrary number of assets and is arbitrage-free. One corollary of that extension is that CIP is respected.

3. Exchange Rate Excess Volatility. In the data, exchange rates are more volatile than fundamentals, a fact often referred to as exchange rate excess volatility. The gamma model helps rationalize this volatility not only by directly introducing new sources of variation, for example, shocks to the risk-bearing capacity of the financiers ($\gamma_1$) and gross flows ($f_t$), but also indirectly by endogenously amplifying fundamental volatility via the financial constraints. The intuition is that higher fundamental volatility tightens financial constraints, tighter constraints lead to higher volatility, thus generating a self-reinforcing feedback loop. We formalize this more subtle effect in the lemma below and sharpen it by not only maintaining the assumption that $\xi_t = 1$ at all dates, but also by considering the case of deterministic ($\gamma_t$), so that the only source of volatility is fundamental and no information revelation about future shocks $E_1[t_2] = E_0[t_2]$ and $\text{Var}_1[t_2] = \text{Var}_0[t_2]$.

**Lemma 5.** (Endogenous Amplification of Volatility). The volatility of the exchange rate at time 1 is $\text{var}(e_1) = \left(\frac{\Gamma_1}{1+\Gamma_1}\right)^2 \text{var}(e_1)$, where $\Gamma_1 = \gamma_1 \text{var}(e_2)$. If $\alpha > 0$ and $\gamma_1 > 0$, then fundamental volatility is endogenously amplified by the financial constraint: $\frac{\partial \text{var}(e_1)}{\partial \text{var}(e_2)} > 0$. Notice that if $\gamma_1 = 0$, then $\frac{\partial \text{var}(e_1)}{\partial \text{var}(e_2)} = 0$.

III.B. Foreign Exchange Rate Intervention

The gamma model of exchange rates considered so far has emphasized the central role of financial forces and in particular
capital flows in the determination of exchange rates. We study one very prominent type of flow: currency intervention by the official sector (the central bank or the treasury department).

Large-scale currency interventions have recently been undertaken by the governments of Switzerland and Israel. Both aimed to relieve their currency appreciation in the face of turmoil in financial markets. By most accounts, the interventions successfully weakened the exchange rate and boosted the real economy. Empirical studies, however, have yet to confront the thorny issue of endogeneity of the policy, and future empirical work is necessary to provide a full empirical assessment.

Here we focus on proving a framework to understand under which conditions foreign exchange rate intervention can be a powerful tool to combat exchange rate movements generated by financial turmoil. The limited risk-bearing capacity of the financiers in our model is at the core of the effects of FX intervention on exchange rates. Indeed, Backus and Kehoe (1989) show that in a general class of models in which currencies are imperfect substitutes due to risk premia, but in which importantly there are no financial frictions, FX interventions have no effect on the exchange rate.

For notational simplicity, we set most parameters at 1: for example, \( \xi_t = a_t = \alpha_0^* = \beta = \beta^* = 1 \). We allow \( \xi_1 \) to be stochastic (keeping \( \mathbb{E}[\xi_1] = 1 \), and setting \( a_1^* = \xi_1 \) for symmetry) so that currency trading is risky.

At time 0, the Japanese government intervenes in the currency market vis-a-vis the financiers: it buys \( q^* \) dollars and sells \( q \) yen. By Proposition 4 we immediately obtain the result.

41. The Czech Republic also intervened in the currency market in November 2013 with the aim of depreciating the koruna to boost the domestic economy.

42. Israel central bank governor Stanley Fisher remarked: “I have no doubt that the massive purchases [of foreign exchange] we made between July 2008 and into 2010 [...] had a serious effect on the exchange rate which I think is part of the reason that we succeeded in having a relatively short recession” (Levinson 2010).

43. Blanchard, de Carvalho Filho, and Adler (2014) find empirical support for the efficacy of this policy. An earlier skeptical empirical literature, which mostly focused on interventions of considerably smaller size, is summarized by Sarno and Taylor (2001). Dominguez and Frankel (1993a,b) find empirical support for the effect of foreign exchange rate intervention via a portfolio balance channel.
below (as the government creates a flow $f^* = q^*$ in the currency market):

**Lemma 6.** If the Japanese government buys $q^*$ dollars and sells $\frac{q^*}{e^0}$ yen at time 0, the exchange rates satisfy: $e_0 = 1 - \frac{\Gamma}{2+\Gamma} q^*$, and $e_1 = 1 + \frac{\Gamma}{2+\Gamma} q^* + \varpi(t1)$, with $\Gamma = \gamma \nu \varpi(t1)^\alpha$.

The intervention has no effect on the average exchange rate: $e_0 + \frac{E[e_1]}{2} = 1$ irrespective of $q^*$. The intervention induces a depreciation at time 0, and an appreciation at time 1. We call this the “boomerang effect.” A currency intervention can change the level of the exchange rate in a given period, but not the average level of the exchange rate over multiple periods. Lemma 6 highlights the importance of the frictions: if $\Gamma = 0$, a frictionless setup analogous to that in Backus and Kehoe (1989), there is no effect of the intervention on the exchange rate. Correspondingly, the potency of the intervention is strictly increasing in the severity of the frictions: the higher the $\Gamma$ the more the exchange rate moves for a given size of the intervention.

A classic criticism of portfolio balance models is that only extremely big interventions are effective because for an intervention to be effective it needs to alter very large stocks of assets: either the entire stock of assets outstanding or the country level gross external assets and liabilities. In our framework interventions are more effective because they need only alter $Q$, the balance sheet of financiers, which is potentially substantially smaller than the entire stock of assets.

Our framework also sheds light on the real consequences of FX intervention on output and risk sharing. We assume that in the short run, that is, period $t = 0$, Japanese tradables’ prices are sticky in domestic currency (PCP) as in Section II.E; prices are flexible in the long run, that is, period $t = 1$. We postulate that at time 0 the price is downward rigid at a level $\bar{P}_F^*$ that is sufficiently high as to cause unemployment in the Japanese tradable sector. U.S. tradable prices are assumed to be flexible. This captures a situation in which one country is in a recession, with high slack capacity and unemployment, so much so that its output is demand driven.

**Proposition 9. (FX Intervention).** Assume that $\Gamma > 0$ and that at time 0 Japanese tradable goods’ prices are downward rigid at a price $\bar{P}_F^*$ that is sufficiently high to cause unemployment in
the Japanese tradable sector. A Japanese government currency intervention, whereby the government buys $q^*$ worth of dollar bonds and sells $\check{q}$ yen bonds at time 0, depreciates the yen and increases Japanese output. $\check{q}$ is the smallest intervention that restores full employment in Japan. The intervention distorts consumption with the consumption shares determined by $C_{H,t} = s^*_t$ and $C_{F,t} = 1 - s^*_t$ with $s^*_t = \frac{e_t}{1 + e_t}$ for $t = 0,1$.

Note that there are two preconditions for this intervention analysis. The first one is that prices are sticky (fixed) in the short run at a level that generates a fall in aggregate demand and induces an equilibrium output below the economy’s potential. This condition, that is, being in a demand-driven state of the world, is central to the Keynesian analysis where a depreciation of the exchange rate leads to an increase in output via an increase in export demand. If this condition is satisfied, a first-order output loss would occur even in a world of perfect finance. The second precondition is that financial markets are imperfect, that is, $\Gamma > 0$. Recall from Lemma 6 that the ability of the government to affect the time 0 exchange rate is proportional to $\Gamma$. When markets are frictionless ($\Gamma = 0$) the government FX policy has no effect on the time 0 exchange rate, even if prices are sticky, because financiers would simply absorb the intervention without requiring a compensation for the resulting risk.

The intervention has two distinct effects on consumption. The first effect is an increase in world consumption because, as already described, the intervention expands Japanese output without decreasing U.S. output. The second effect is a distortion in the share of world output consumed by each country. Both effects are clearly illustrated by the Japanese consumption of Japanese tradables: $^{44}$

$$C^*_{F,0} = \frac{1}{p^*_F} = \frac{e_0}{1 + e_0} \frac{Y_{F,0}}{L}.$$ 

The term $\frac{e_0}{1 + e_0}$ is the equilibrium share of Japanese tradables consumed by Japanese households. The intervention reduces

$^{44}$ The first equality follows from the demand function of Japanese households for Japanese tradables ($C_{F,t} = \frac{q_t}{p_{F,t}}$), the second equality follows from the equilibrium output function of Japanese tradables in Section II.E.
this share via a dollar appreciation ($e_0 \downarrow$). At the same time, the intervention increases total Japanese output by reducing slack: the term $\frac{Y_{F,0}}{L} \in (0, 1]$ is decreasing in $e_0$ since output is demand driven. The functional specifications of the model (logarithmic utility and linear production) make the two effects cancel out and keep $C_{F,0}^s$ unchanged; the boomerang effect, however, will induce an expected increase in the consumption next period ($\mathbb{E}[C_{F,1}^s] \uparrow$). U.S. consumption of both tradable goods increases at time 0, due to both an increase in U.S. share of world consumption and an increase in output, but then falls at time 1.

Overall the intervention boosts world output with the cost of intertemporal distortions in consumption. Interestingly, the suggested policy is not of the “beggar thy neighbor” type: the Japanese currency intervention, even with its aim to weaken the yen, actually increases consumption, at least in the short run, in the United States. The United States benefits from an increase in Japanese output with no loss of U.S. output. We highlight that currency wars can only occur when both countries are in a slump and the post-intervention weaker yen causes a first-order output loss in the United States.\textsuperscript{45}

The intervention has real effects even in this calibration that has been chosen so that before the intervention households have no incentives to trade in financial markets even if they were allowed to do so freely and optimally. Similarly, the financiers have no incentives to trade at equilibrium prices before and after the intervention.\textsuperscript{46} To further isolate the sole effect of financial frictions on the intervention outcome, we assumed that the

\textsuperscript{45} Cavallino (2014) builds on this analysis and analyzes the joint use of FX intervention and monetary policy.

\textsuperscript{46} Indeed, in this economy (before the government intervention) the exchange rate is at 1, and is expected to remain at 1 on average, therefore financiers optimally choose to not trade at all. Similarly, U.S. households are on their “shadow” Euler equation and would not want to trade yen bonds even if allowed to do so. Japanese households would have a small incentive to trade since their shadow Euler equation has a Jensen inequality term (an additional term compared to the U.S. household Euler equation). After the intervention, financiers still have no further incentives to trade having already optimized their positions in response to the intervention. Of course, households would now like to trade but these unlimited optimal trades are not possible (here as in the rest of the model) due to the frictions in the intermediation process. Therefore the policy success relies on the presence of financial frictions rather than a direct failure of Ricardian equivalence.
intervention’s proceeds and losses are rebated lump sum (i.e., non-distortionary) by the Japanese government to its citizens.

1. The Potency of Intervention: Combining FX Intervention and Capital Controls. It is often argued by policy makers that currency intervention should be undertaken together with capital controls. The gamma model provides a unified view of this policy combination because capital controls increase the financial market segmentation thus enhancing the potency of currency intervention.

We introduce a second policy instrument, taxation of the financiers, which is a form of capital controls. We consider a proportional (Japanese) government tax on each financier’s profits; the tax proceeds are rebated lump sum to financiers as a whole. Recall the imperfect intermediation problem in Section II.B; we now assume that the after-tax value of the intermediary is \( V_t(1 - \tau) \), where \( \tau \) is the tax rate. The financiers’ optimality condition, derived in a manner entirely analogous to the optimization problem in equation (9), is now \( Q_0 = \frac{\mathbb{E} [e_0 - e_1 g_1]}{\Gamma \tau (1 - \tau)} \). Notice that this is equivalent to changing \( \Gamma \) to an effective \( \Gamma_{\text{eff}} = \frac{\Gamma}{\Gamma \tau (1 - \tau)} \), so that the financiers’ demand can be rewritten as \( Q_0 = \frac{\mathbb{E} [e_0 - e_1 g_1]}{\Gamma_{\text{eff}} \tau (1 - \tau)} \). We consider the leading case of \( \xi_t \) deterministic and collect the result in the proposition below.

**Proposition 10.** Assume \( \xi_t \) is deterministic, a tax \( \tau \) on finance is equivalent to lowering the financiers’ risk-bearing capacity by increasing \( \Gamma \) to \( \Gamma_{\text{eff}} = \frac{\Gamma}{\Gamma \tau (1 - \tau)} \). A higher tax increases the effective \( \Gamma_{\text{eff}} \), thus reducing the financiers’ risk-bearing capacity. The sign of the effect of the tax on exchange rates depends on the position that the financiers would have taken absent the tax (\( Q_0 \)): if the financiers were long (short) dollars \( Q_0 > 0 \) (\( Q_0 < 0 \)), then a tax depreciates (appreciates) the dollar at time \( t=0 \). The potency of FX intervention increases in the tax.

47. There is a recent and interesting literature on the use of capital controls: Bianchi (2010); Mendoza (2010); Korinek (2011); Magud, Reinhart, and Rogoff (2011); Farhi and Werning (2012a,b, 2013, 2014); Schmitt-Grohé and Uribe (2012); Rey (2013); Costinot, Lorenzoni, and Werning (2014); Farhi, Gopinath, and Itskhoki (2014).
First we note that if the equilibrium before the government intervention features zero risk taking by the financiers \((Q_0^C = 0)\), as was the case in the economy studied in the previous analysis of FX intervention, then the tax \(\tau\) is entirely ineffective. Intuitively, this occurs because there are zero expected profits to tax, and therefore the tax has no effect on ex-ante incentives.

More generally we recall from Proposition 2 that an increase in \(\Gamma\), in this case an increase in \(\Gamma^{\text{eff}}\) due to an increase in \(\tau\), has the opposite effect on the exchange rate depending on whether the financiers are long or short the dollar to start with, that is, depending on the sign of \(Q_0^C\) before the tax is imposed. For example, the tax would make the dollar depreciate on impact if the financiers were long dollars to start with \((Q_0^C > 0)\), but the same tax would make the dollar appreciate if the financiers had the opposite position to start with. In practice this means that policy makers who are considering imposing capital controls, or otherwise taxing international finance, should pay close attention to the balance sheets of financial institutions that have exposures to their currency. Basing the policy on reduced-form approaches or purely on traditional macroeconomic fundamentals not only can be misleading but might actually generate the opposite outcome for the exchange rate from the desired one. Finally, recall from Lemma 6 that the effect of currency intervention on the exchange rate is bigger the lower the financiers’ risk-bearing capacity (the higher the \(\Gamma\)). It follows that a tax on finance or a capital control, by implicitly reducing risk-bearing capacity, increases the potency of FX intervention.

IV. ANALYTICAL GENERALIZATION OF THE MODEL

The basic version of the gamma model presented so far was real, and we now show that it can readily be extended to a nominal version where the nominal exchange rate is determined, similarly to our baseline model, in an imperfect financial market.\(^{48}\)

\(^{48}\) Notice that we have indeed set up the “real” model in the main text in such a way that nontradables in each country play a role very similar to money and where therefore the exchange rate is rather similar to a nominal exchange rate (see Obstfeld and Rogoff 1996, chap. 8.3). In this section we make such analogy more explicit. Online Appendix A.1.D provides a full discussion of the CPI-based real exchange rate and the nominal exchange rate in our model. Alvarez, Atkeson, and Kehoe (2009) provide a model of nominal exchange rates with frictions in the
We assume that money is only used domestically by the households and that its demand is captured, in reduced form, in the utility function of households in each country. Financiers do not use money, but they trade in nominal bonds denominated in the two currencies. The U.S. consumption basket is now extended to include a real money balances component such that the consumption aggregator is

\[ C_t \equiv \left( \frac{M_t}{P_t} \right)^{\omega_t} (C_{NT,t})^{\chi_t} (C_{H,t})^{\alpha_t} (C_{F,t})^{\gamma_t} \frac{1}{R} \]

where \( M \) is the amount of money held by the households and \( P \) is the nominal price level so that \( M \times P \) is real money balances. We maintain the normalization of preference shocks by setting \( \theta_t \equiv \omega_t + \chi_t + \alpha_t + \gamma_t \). Correspondingly, the Japanese consumption basket is now

\[ C_t^* \equiv \left( \frac{M_t}{P_t} \right)^{\omega_t} (C_{NT,t})^{\chi_t} (C_{H,t})^{\alpha_t} (C_{F,t})^{\gamma_t} \frac{1}{R} \]

Money is the numéraire in each economy, with local currency price equal to 1. The static utility maximization problem is entirely similar to the one in the basic gamma model in Section II.A, and standard optimization arguments lead to demand functions:

\[ M_t = \frac{\omega_t}{\lambda_t}; \quad p_{NT,t}C_{NT,t} = \frac{\mu_t}{\lambda_t}; \quad p_{F,t}C_{F,t} = \frac{\nu_t}{\lambda_t}, \]

where, we recall from earlier sections, \( \lambda_t \) is the Lagrange multiplier on the households’ static budget constraint. Substituting for the value of the Lagrange multiplier, money demand is given by \( M_t = \omega_t P_t C_t \) and is proportional to total nominal consumption expenditures; the coefficient of proportionality, \( \omega_t \), is potentially stochastic.

49. A vast literature has focused on foundations of the demand for money; such foundations are beyond the scope of this article and consequently we focus on the simplest approach that delivers a plausible demand for money and much tractability.

50. The budget constraint of the households is now:

\[ \sum_{t=0}^{1} R^{-t} \left( p_{NT,t} Y_{NT,t} + p_{H,t} Y_{H,t} + M_t \right) = \sum_{t=0}^{1} R^{-t} \left( p_{NT,t} C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t} + M_t \right) \]

where \( M_t \) is the seigniorage rebated lump sum by the government, which is equal to \( M_t \) in equilibrium.

51. The money demand equation is similar to that of a cash in advance constraint where money is only held by the consumers within the period, that is, they need to have enough cash at the beginning of the period to carry out the planned period consumption. For constraints of this type see Helpman (1981) and Helpman and Razin (1982).
Let us define $m_t \equiv \frac{M_t^0}{\omega_t}$ and $m_t^* \equiv \frac{M_t^0*}{\omega_t^*}$, where $M_t^0$ and $M_t^0*$ are the money supplies.\(^{52}\) Notice that since money (as in actual physical bank notes) is nontradable across countries or with the financiers (but bonds that pay in units of money are tradable with the financiers as in the previous sections), the money market clearing implies that the central bank can pin down the level of nominal consumption expenditure ($m_t = \lambda_t^{-1}, m_t^* = \lambda_t^*-1$).\(^{53}\) The nominal exchange rate $e_t$ is the relative price of the two currencies. It is defined as the strength of the yen, so that an increase in $e_t$ is a dollar depreciation.\(^{54}\)

U.S. nominal imports in dollars are $p_{F,t} C_{F,t} = \xi_t = \iota_t m_t$. Similarly, Japanese demand for U.S. tradables is $p_{H,t}^* C_{H,t}^* = \xi_t m_t^*$. Hence, U.S. nominal exports in dollars are: $p_{H,t}^* C_{H,t}^* e_t = \xi_t e_t m_t^*$. We conclude that U.S. nominal net exports in dollars are $NX_t = \xi_t e_t m_t^* - \iota_t m_t$.

We assume that the financiers solve: \(^{55}\)

$$\max_{q_0} V_0 = \Omega_0 q_0, \quad \text{subject to } V_0 \geq \min \left(1, \Gamma \frac{|q_0|}{m_0^* e_0}\right) |q_0|,$$

where $\Omega_0 = \mathbb{E}_0 \left[1 - \frac{R^* e_1}{R e_0}\right]$. Notice that $m_0^*$ is now scaling the portion of nominal assets that the financiers can divert to ensure that such fraction is scale

\(^{52}\) It is often convenient to consider the cashless limit of our economies by taking the limit case when $m$ and $m^*$ are policy variables. We abstract here from issues connected with the zero lower bound on nominal interest rates. Notice the duality between money in the current setup and nontradable goods in the basic gamma model of Section II. If $M_t = \omega_t$ and $C_{NT,t} = \chi_t$, one recovers the equations in Section II, because the demand for money implies $\lambda_t = 1$, in which case the demand for nontradables implies that $p_{NT,t}^* = 1$.

\(^{53}\) The central bank in each period chooses money supply after the preference shocks are realized so that $m$ and $m^*$ are policy variables. We abstract here from issues connected with the zero lower bound on nominal interest rates. Notice the duality between money in the current setup and nontradable goods in the basic gamma model of Section II. If $M_t = \omega_t$ and $C_{NT,t} = \chi_t$, one recovers the equations in Section II, because the demand for money implies $\lambda_t = 1$, in which case the demand for nontradables implies that $p_{NT,t}^* = 1$.

\(^{54}\) To keep simpler notations, we denote the nominal exchange rate by $e_t$, the same symbol used for the exchange rate in the basic gamma model.

\(^{55}\) When we consider setups that are more general than the basic gamma model of Section II, we maintain the simpler formulation of the financiers’ demand function. We do not directly derive the households’ valuation of currency trades in these more general setups. Our demand functions are very tractable and carry most of the economic content of more general treatments; we leave it for the extension Section A.4 to characterize numerically financier value functions more complex than those analyzed in closed form here.
invariant to the level of the Japanese money supply and hence the nominal value in yen of the assets.56

Finally, the nominal interest rates are given by the households’ intertemporal optimality conditions (Euler equations):

$$1 = \mathbb{E} \left[ \beta R \frac{U_1^{'2} C_{NT.1} P_{NT.0}}{U_0^{'2} C_{NT.0} P_{NT.1}} \right] = \mathbb{E} \left[ \beta R \frac{\bar{x}_1}{\bar{z}_0} C_{NT.1} P_{NT.0} \right] = \beta R \mathbb{E} \left[ \frac{m_0}{m_1} \right],$$

so that $R^{-1} = \beta \mathbb{E} \left[ \frac{m_0}{m_1} \right]$. Similarly, $R^{-1*} = \beta^* \mathbb{E} \left[ \frac{m_0}{m_1} \right]$. These interest rate determination formulas extend those in equation (6) to the nominal setup.

IV.A. Equilibrium Exchange Rate in the Extended Setup

When we include all the extensions to the basic gamma model considered so far, the key equations to solve for the equilibrium nominal exchange rate are the flow equations in the international bond market:

$$m^*_0 e_0 - m_0 t_0 + Q_0 + f^* - f e_0 - D^U = D^f e_0 = 0, \tag{25}$$

$$m^*_1 e_1 - m_1 t_1 - R Q_0 - R^* f^* + R^* f e_1 = 0, \tag{26}$$

and the financiers’ demand curve:

$$Q_0 = \frac{m_0^*}{\Gamma} \mathbb{E} \left[ e_0 - e \frac{R^*}{R} \right]. \tag{27}$$

Equations (25)–(26) allow for household trading of foreign currency. They extend Section II.D.1, which only considered liquidity/noise trading, by allowing these demand functions for foreign bonds to depend on all fundamentals but not directly on the exchange rate.57 Equations (25)–(26) also allow for each

56. The constraint $\Gamma = \gamma_{var}(e_1^a)$ can become $\Gamma = \gamma_{var} \left( e_1^a \frac{m_0}{m_1} \right)^a$, to make the model invariant to predictable changes to money supply.

57. We allow the demand functions for foreign bonds from U.S. and Japanese households, denoted by $f$ and $f^*$ respectively, to depend on all present and expected future fundamentals. We use the shorthand notation $f$ and $f^*$ to denote the generic functions: $f(R, R^*, t, \xi, ...)$. For example, demand functions that load on a popular trading strategy, the carry trade, that invests in high interest rate currencies while funding the trade in low interest rate currencies can be expressed as $f = b + c(R - R^*)$ and $f^* = d + g(R - R^*)$, for some constants $b, c, d, g$. Interestingly, both gross capital flows and trade flows could be ultimately generated by financial frictions (see Antrás and Caballero 2009). Dekle, Hyeok, and Kiyotaki (2014) employ the reduced-form approach and put holdings of foreign
country to start with a stock of foreign assets and liabilities. The U.S. net foreign liabilities in dollars are $D^{US}$ and Japan net foreign liabilities in yen are $D^{J}$.58

We show in the proposition below that the solution method, even in this more general case, follows the simple derivation of the basic model by representing the current economy as a “pseudo” basic economy. We also note that these results do not impose that $Y_{NT,t} = x_t$ and $Y_{NT,1} = x_t^*$, thus generalizing the analysis in Section II.

**Proposition 11.** In the richer model above (with money, portfolio flows, external debt, and shocks to imports and exports) the values for the exchange rates $e_0$ and $e_1$ are those in Proposition 3, replacing imports $(i_t)$, exports $(x_t)$, and the risk-bearing capacity $(\Gamma)$ by their pseudo counterparts $[\tilde{i}_t, \tilde{x}_t, \tilde{\Gamma}]$, defined as: $	ilde{i}_0 = m_0 t + D^{US} - f^*; \tilde{x}_0 = m^* t + D^{J} - f; \tilde{i}_1 = m_{11} t + Rf^*; \tilde{x}_1 = m_{1} t + Rf^*; \tilde{\gamma} = \frac{\gamma}{m_{0}}; \tilde{v} = \frac{\gamma}{m_{0}}; \tilde{\Gamma} = \frac{\gamma}{m_{0}}$.

**Proof.** Equations (25)–(26) reduce to the basic flow equations, equations (13)–(14), provided we replace $i_t$ and $x_t$ with $\tilde{i}_t$ and $\tilde{x}_t$. Similarly, equation (27) reduces to equation (10), provided we replace $\Gamma$ with $\tilde{\Gamma}$. Then the result follows from the proof of Proposition 3. □

Intuitively, the pseudo imports $(\tilde{i})$ are composed of factors that lead consumers and firms to sell dollars and hence “force” financiers to be long the dollar. An entirely symmetric intuition applies to the pseudo exports $(\tilde{\xi})$.

We collect a number of qualitative results for the generalized economy. While some properties do not strictly depend on $\Gamma > 0$ and therefore can be derived even in UIP models, it is nonetheless convenient to provide a unified treatment in the present model. We assume that $\tilde{i}_t$ and $\tilde{x}_t$ are positive at dates 0 and 1. Otherwise, various pathologies can happen, including the nonexistence of an equilibrium (e.g., formally, a negative exchange rate).

58. We could have alternatively assumed that only a fraction $\eta$ of the debt had to be intermediated in which case we would get a flow of $\eta D$ at time 0 and a flow $(1 - \eta)RD$ at time 1.
PROPOSITION 12. The dollar is weaker:

(i) (Imports-exports) when U.S. import demand for Japanese goods \( (t_i) \) is higher; when Japanese import demand for U.S. goods \( (\xi_t) \) is lower;

(ii) (“Myopia” from an imperfect financial system) higher \( \Gamma \) increases the effects in point (i) by making current imports matter more than future imports;

(iii) (Debts and their currency denomination) when U.S. net external liabilities in dollars \( (D^{US}) \) are higher; when Japanese net external liabilities in yen \( (D^J) \) are lower;

(iv) (Financiers’ risk-bearing capacity) when financial conditions are worse \( (\Gamma \text{ is higher}) \), conditional on Japan being a net creditor at time \( 0^+ (N_0^+ < 0) \);

(v) (Demand pressure) when the noise demand for the dollar \( (f^*) \) is lower, as long as \( \Gamma > 0 \);

(vi) (Interest rates) when the U.S. real interest rate is lower; when the Japanese real interest rate is higher;

(vii) (Money supply) when the U.S. current money supply \( (m_0) \) is higher; when the Japanese current money supply \( (m_0^J) \) is lower.

Point (iii) highlights a valuation channel to the external adjustments of countries. The exchange rate moves in a way that facilitates the reequilibration of external imbalances. Interestingly, it is not just the net external position of a country, its net foreign assets, that matters for external adjustment, but actually the (currency) composition of its gross external assets and liabilities \( (D^{US} \text{ and } D^J) \). This basic result is consistent with the valuation channel to external adjustment highlighted in Gourinchas and Rey (2007a) and Lane and Shambaugh (2010).

V. CONCLUSION

We presented a theory of exchange rate determination in imperfect capital markets where financiers bear the risks resulting from global imbalances in the demand and supply of international assets. Exchange rates are determined by the balance

59. That is, \( \frac{\partial e}{\partial \xi_t} > 0 \) and \( \frac{\partial e}{\partial t_i} > 0 \) are positive and respectively increasing and decreasing in \( \Gamma \).
sheet risks and risk-bearing capacity of these financiers. Exchange rates in our model are disconnected from traditional macroeconomic fundamentals, such as output, inflation, and the trade balance and are instead more connected to financial forces such as the demand for assets denominated in different currencies. Our model is tractable, with simple-to-derive closed-form solutions, and can be generalized to address a number of both classic and new issues in international macroeconomic analysis.

APPENDIX

The financiers’ optimization problem.

We clarify the role of the mild assumption, made in footnote 15, that \( 1 \geq \Omega_0 \geq -1 \). Formally, the financiers’ optimization problem is:

\[
\max_{q_0} V_0 = \Omega_0 q_0, \quad \text{subject to } V_0 \geq \min \left( 1, \Gamma \frac{|q_0|}{e_0} \right)|q_0|,
\]

where \( \Omega_0 \equiv E \left[ 1 - \frac{R^* e_1}{R e_0} \right] \).

Notice that \( \Omega_0 \) is unaffected by the individual financier’s decisions and can be thought of as exogenous in this constrained maximization problem.

Consider the case in which \( \Omega_0 > 0 \), then the optimal choice of investment has \( q_0 \in (0, \infty) \). Notice that \( \Omega_0 \leq 1 \) trivially. Then one has \( V_0 \leq q_0 \). In this case, the constraint can be rewritten as \( V_0 \geq \Gamma \frac{q_0}{e_0} \), because the constraint will always bind before the portion of assets that the financiers can divert \( \Gamma \frac{|q_0|}{e_0} \) reaches 1. This yields the simpler formulation of the constraint adopted in the main text.

Now consider the case in which \( \Omega_0 < 0 \), then the optimal choice of investment has \( q_0 \in (-\infty, 0) \). It is a property of currency excess returns that \( \Omega_0 \) has no lower bound. In this article, we assume that the parameters of the model are such that \( \Omega_0 > -1 \), that is, we assume that the worst possible (discounted) expected returns from being long a dollar bond and being short a yen bond is -100 percent. Economically this is an entirely innocuous assumption given that the range of expected excess returns
in the data is approximately [−6 percent, +6 percent]. With this assumption in hand we have \( V_0 \leq |q_0| \), and hence we can once again adopt the simpler formulation of the constraint because the constraint will always bind before the portion of assets that the financiers can divert \( \Gamma \frac{|q_0|}{\pi} \) reaches 1.

As pointed out in the numerical generalization section of the Online Appendix (Section A.4), more general (and nonlinear) value functions would apply once the simplifying assumptions made in the text are removed and depending on to whom the financiers repatriate their profits and losses. In the main body of the article, we maintain the assumption that the financiers use the U.S. household valuation criterion; this makes the model most tractable while very little economic content is lost. The numerical generalizations in the Online Appendix provide robustness checks by solving the nonlinear cases.

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SUPPLEMENTARY MATERIAL

An Online Appendix for this article can be found at QJE online (qje.oxfordjournals.org).

REFERENCES


